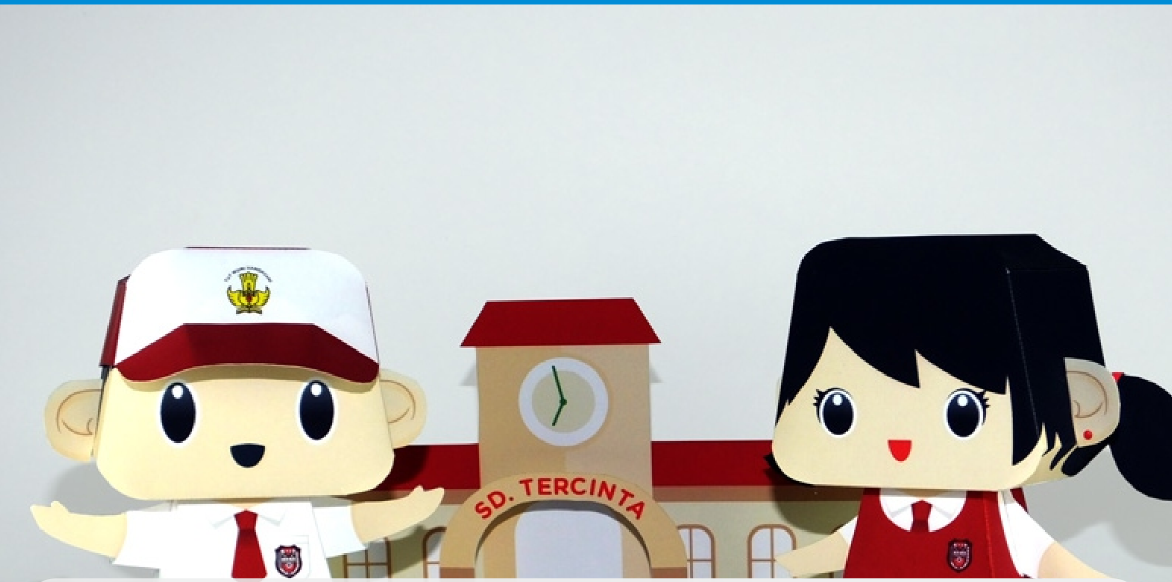
Fajar Arwadi | Bustang | Ratu Ilma Indra Putri | Somakim



Small-Scale Design-Based Research on Elementary School Children's Skills and Understanding of Combinatorics: A Case of Indonesia

Global Research and Consulting Institute (Global-RCI)

UNDANG-UNDANG REPUBLIK INDONESIA
NOMOR 28 TAHUN 2014
TENTANG HAK CIPTA

PASAL 113
KETENTUAN PIDANA

(1) Setiap orang yang dengan tanpa hak melakukan pelanggaran hak ekonomi sebagaimana dimaksud dalam Pasal 9 ayat (1) huruf i untuk Penggunaan Secara Komersial dipidana dengan pidana penjara paling lama 1 (satu) tahun dan/atau pidana denda paling banyak Rp. 100.000.000,00 (seratus juta rupiah).
(2) Setiap orang yang dengan tanpa hak dan/atau tanpa izin Pencipta atau pemegang Hak Cipta melakukan pelanggaran hak ekonomi Pencipta sebagaimana dimaksud dalam Pasal 9 ayat (1) huruf c, huruf d, huruf f, dan/atau huruf g untuk Penggunaan Secara Komerial dipidana dengan pidana penjara paling lama 3 (tiga) tahun dan/atau pidana denda paling banyak Rp. 500.000.000,00 (lima ratus juta rupiah).
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(4) Setiap orang yang memenuhi unsur sebagaimana dimaksud pada ayat (3) yang dilakukan dalam bentuk pembajakan, dipidana dengan pidana penjara paling lama 10 (sepuluh) tahun dan/atau pidana denda paling banyak Rp. 4.000.000.000,00 (empat miliar rupiah)

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Fajar Arwadi
Bustang
Ratu Ilma Indra Putri
Somakim

2017

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Penulis : Fajar Arwadi, Bustang, Ratu Ilma Indra Putri, Somakim

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Penyunting	:	Muhammad Yusran Basri
Perancang Sampul	:	Muhammad Iswan Achlan
Penata Letak	:	Muhammad Yusran Basri
Isi	:	Sepenuhnya tanggung jawab penulis

Diterbitkan Oleh:

Global Research and Consulting Institute (Global-RCI)
Kompleks Alauddin Business Center (ABC)
Jalan Sultan Alauddin No. 78 P, Makassar, Indonesia, 90222. Telepon: 08114100046, Homepage: http://www.global-rci.com.

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PREFACE

The present book is made from the sequence of research activities conducted by the authors. As most research reports, it sequentially consists of five chapters. It describes the introduction in the beginning consisting of the background why the research was administered and the research question. The next chapter outlines several supporting literatures for the research method including discrete mathematics and combinatorics, constructivism learning, and realistic mathematics education. The research method is explained in the chapter three which mainly comprehends of design research and hypothetical learning trajectory. The fourth chapter details the findings of the research as well as the discussion before drawing the conclusion in the chapter five.

The authors hope this book is certainly useful for everyone, particularly for teachers in elementary school children, lecturers, and researchers who aim to develop the research further. However, critiques and advices are emphatically needed for the refinement of this book in the future.

Author

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CHAPTER I

INTRODUCTION

Problem solving is one of the issues in mathematics education developed by, one of them, NCTM in 1980's decade (Mathematics, 1980). Since one of the natures of the problem solving is confronting novel situation (Szetela & Nicol, 1992), students are expected to use their own knowledge and strategies, not relying on applying algorithm or mathematics formulas in a textbook for solving the problem. It makes sense since it is supported by several types of research that suggest novel problem solving for children since it believes that children can gain new knowledge from their own experimentations (Gelman & Brown, 1986). In addition, Vygotsky (1978) stated a theory i.e. zone of proximal development that is a measure which determines a distance of which children are able to solve a question or problem with or without assistance from others. He also suggested that social interaction is needed to help students in extending their problem-solving competencies without assistance. On the other hand, (Brown & Reeve, 1987) claimed that students are able to broaden their own problem solving competencies without assistance if there is no external intervention when they are given opportunity to solve problems. The suggestion of Brown and Reeve is asserted by (English, 1996) that children are able to solve novel problems which are more sophisticated for them.

As one of the branches of mathematics, discrete mathematics broadly has served other disciplines such as computer science, engineering, statistics and probability, etc. It causes the problems of discrete mathematics found in many curricula are mostly in the form of applied mathematics and more familiar for both children and adult compared to some other branches of mathematics. It is then interesting to use such topic as the material for problem solving in mathematics since one can set a problem which is closely related to children's daily life and challenging for them to solve. In realistic mathematics education, the problem can be considered as context as a path aimed to grasp mathematical concepts (Bakker, 2004).

One of the topics in discrete mathematics which gets major representation in school curriculum is combinatorics (Kavousian, 2008). Such kind of development implied educational studies in that topic also quite evolve. In Indonesia curriculum, the combinatorics topic is studied firstly in senior high school level. It mainly covers multiplication principle, factorial, permutation, and combination. However, there are several studies ((English, 2007), (Halani, 2012), (Höveler, 2014),;(Piaget & Inhelder, 2014)) suggesting that combinatorics can be introduced in elementary level. Besides that, it is supported by Vygotsky (1978) i.e. zone of proximal development that is a measure which determines the gap of which a child can solve problems with or without the help of others. He also suggested that social interaction of students is necessary for them in extending their problem-solving competencies without assistance. In addition, Brown & Reeve (1987) claimed that students can extend their competencies without assistance when they are given opportunity to solve novel problems. It is also suggested by English (1996) that children are able to solve novel problems which are more sophisticated for them. Moreover, English (2007), Yuen (2008), and Höveler (2014) have respectively studied the strategies used by elementary school children in solving combinatorics problems and the relationship between students' strategies and mathematical counting principles. Meanwhile, the present study is like combining the three latest studies with few differences. Firstly, it designs learning activities in constructivism approach to facilitate elementary school children skills by using the efficient strategies by English (2007) in solving combinatorics problems related to multiplication principle. It used constructivism approach based on the philosophy of (Davis, 1990), that learners have to construct their own knowledge both individually and collectively especially from solving problems. Likewise, it has positive effect on students' learning (Hmelo-Silver, Duncan, & Chinn, 2007); (Nayak, 2007); (Monoranjan, 2015). Secondly, it connects the strategies to construct students' conception of the topic

In specific to the research of the studies of students' strategies of English (2007) covering multiplication principle problem, there are some strategies used by elementary school children in solving two-dimensional problem and three-dimensional problem highlighted. In addition, the efficiency of those strategies is also emphasized. The *trial and error* approach and the *odometer* pattern were respectively considered as the most inefficient and efficient strategy. The latter strategy which was named since it resembles the odometer in a vehicle is conceptually and closely related to the multiplication concept since if there are *m* items in each *n* and there are *n* items, then there will be *m* multiplied by *n* items in total. Meanwhile, in the three-dimensional strategy, the most useful strategy to the concept formation is *major-minor.* It is so labeled since there is a *major* item which is less frequently changed and paired to each minor item. These efficient strategies also definitely represent the concept of multiplication as the introductory part of the combinatorial topic which is mostly studied in secondary level.

What makes the present research different to some previous researches is that it designs learning activities in constructivism approach to facilitate elementary school children skills by using the efficient strategies by English (2007) in solving combinatorics problems related to multiplication principle. It used constructivism approach based on the phylosophy of Davis (1990), that learners have to construct their own knowledge both individually and collectively especially from solving problems. Likewise, it has positive effect on students' learning ((Hmelo-Silver et al., 2007); (Monoranjan, 2015); (Nayak, 2007)). Besides that, the other difference is that it connects the strategies to construct students' conception of the topic. Considering the potency of students in extending their competencies in problem solving, the novelty of the topic for elementary school children, the rarity of studies and the need of guiding them to comprehend the concept of multiplication principle, hence, the researchers are interested to design a learning of which it formulates a sequence of activities to assist children to apply those efficient strategies and to grasp the multiplication concept. Then, the present research question was posed: *how can the designed learning activities support elementary school children to apply efficient strategies in solving problem as well as to reach the understanding of multiplication principle concept*?

Hence, the research set objective, i.e. designing a learning instruction consisting of a sequence of activities to lead students to the desired strategies and the understanding of multiplication principle as learning goals. In addition, it aimed to create learning packages to obtain the goals.

CHAPTER II

LITERATURE REVIEW

In this chapter, several supporting literatures for the explanation of some terms related to this study as well as the basis of designing learning are quite comprensively described. However, the other literatures which are explicitly used to design the students' activities and the teacher guide are concerned in some next chapters

A. Discrete Mathematics and Combinatorics

Discrete mathematics refers to a branch of mathematics dealing with discrete objects, i.e. objects which can be separated from each other. Integers, tables, chairs, students are all discrete objects. On the other hand, real numbers which include irrational as well as rational numbers are not discrete. Since any two different real numbers there is another real number different from either of them. So, they are packed without any gaps and cannot be separated from their immediate neighbors. The typical topics but not limited to are graph theory, discrete optimization, and counting techniques. There are several important reasons for studying discrete mathematics. Firstly, students can develop their ability to understand and create mathematical arguments. In addition, students will simplify themselves in understanding mathematical sciences. Second, discrete mathematics is the gateway to more advanced courses in all parts of the mathematical sciences. Discrete mathematics provides the mathematical foundations for many computer science courses including data structures, algorithms, data base theory, automata theory, formal languages, compiler theory, computer security, and operating systems.

One of the major topics in discrete mathematics is combinatorics which is one of the issues which is very closely related to other disciplines, e.g. computer science, biology, physics, chemistry, and others. Typically, combinatorics deals with finite structures such as graphs, hypergraphs, partitions or partially ordered sets. However, rather than the object of study, what characterizes combinatorics are its methods: counting arguments, induction, inclusion-exclusion, the probabilistic method - in general, surprising applications of relatively elementary tools, rather than gradual development of a sophisticated machinery. That is what makes combinatorics very elegant and accessible, and why combinatorial methods should be in the toolbox of any mainstream mathematician. One of the topics in combinatorics which is popular for students in middle school is factorial, permutation, and combination. Before studying such topics, multiplication is taught for basis of counting principle conception.

B. Constructivism Learning

It is undeniable, most Indonesia's teachers use direct teaching model. Such kind of model puts knowledge as the thing which is passively received either through the senses or by way of communication (Von Glasersfeld, 1990). It is appropriate to the traditional mathematics instruction and curricula which are based on the transmission, or absorption, in view of teaching and learning. In this view, students passively "absorb" mathematical structures which invented by others an recorded in texts or known by authoritative adults. The meaning of constructivism varies according to one's perspective and position. Within educational contexts there are several philosophical meanings of constructivism, as well as personal constructivism as described by Piaget (1967), social constructivism outlined by Vygotsky (1978), and radical constructivism advocated by Von Glasersfeld (1995). Social constructivism and educational constructivism (including theories of learning and pedagogy) have had the greatest impact on instruction and curriculum design because they seem to be the most conducive to integration into current educational approaches. Within constructivist theory, knowledge isn't something that exists outside of the learner. According to Tobin & Tippins (1993), constructivism is a form of realism where reality can only be known in a personal and subjective way.

Mathematics is clearly one of lessons which can cause negative experience for children. If a child has negative experience in mathematics, that experience would affect his / her achievement as well as attitude towards mathematics during adulthood. The obvious question is whether students' failure to learn mathematics can be ascribed to problems of curriculum, problem of teaching, or the student, or perhaps the combination of these (Carnine, 1997). There are many possible reasons as to why students fail in mathematics. But most of the reasons are related to curriculum and methods of teaching rather than the students' lack of capacity to learn (Carnine, 1997). Airasian & Walsh (1997) argue that the existing mode of teaching of mathematics in schools has not fulfilled the needs of the vast majority of our students, and that not nearly enough instructional stress is put on the higher order skills. Traditional method of teaching makes the learner to memorize information, conduct well organized experiments and perform mathematical calculations using a specific algorithm and makes them submissive and rule-bound. The traditional teacher as information giver and the textbook guided classroom have failed to bring about the desired outcomes of producing thinking students (Young & Collin, 2004). A much heralded alternative is to change the focus of the classroom from teacher dominated to student-centred using a Constructivist Approach. Constructivist teaching practices in Science and Mathematics classrooms are intended to produce much more challenging instruction for students and thus, produce improved meaningful learning. These changes have led to instruction in which students are expected to contribute actively to mathematics lessons by explaining their mathematical reasoning to each other and constructing their own understanding of mathematical concepts. Research has shown such a constructivist-based approach to be promising (Ginsburg-Block & Fantuzzo, 1998), and its positive effects have been found for both students' performance and motivation. Such constructivist instruction appears to motivate students because they find it more pleasant to learn and more challenging to study in the constructivist classroom (Ames & Ames, 1985). Constructivist pedagogy is a meta-learning strategy that can be used to develop students' capacity to learn mathematics independently.

C. Realistic Mathematics Education

The choosing of Realistic Mathematics Education (RME) as the approach in designing learning of this study is based by its functions which not only offers a pedagogical and didactical philosophy on teaching and learning mathematics but also designing instructional materials for learning (Bakker, 2004). In addition, it is used as a means of encouraging students to invent their mathematics (Dickinson and Hough, 2012) which fits to the nature of constructivism. Moreover, Freudenthal (2006) stated that since one of the characteristics of RME which allows students to invent their own strategies in solving problems and leads students to gain the goal of learning i.e. understanding mathematics concept, RME-based research fits the research question, e.g., posed in this study. The stage of RME crucially highlighted is that how to support students in reaching mathematical concept understanding stemming from their own strategies using a model by the guide of teachers (Dickinson & Hough, 2012). Bakker (2004) suggested that the model itself is a representation made by the situation of the problem given in which there is a mathematical concept. In this study, the researchers attempted to create the guide by creating the activities of which students use the desired efficient strategies as the model and come up with the multiplication as the mathematics concept. The designed learning activity would also apply the tenets of RME (Bakker, 2004), i.e. using context from the outset, using students' own productions, and promoting the interactivity among students to let them freely discuss what have they made.

CHAPTER III

RESEARCH METHOD

As in this study, a sequence of activities to support students' comprehending and skills was designed, design research then was chosen as the method of the research. Gravemeijer and Cobb (2006) stated there are three phases of design research: the preparation for the experiment, the classroom experiment, and the restropective analyses. In the preparation phase, a hypothetical learning trajectory (HLT) was designed which comprehends of learning goals, teaching and learning activities, and conjecture of student's thinking (Bakker, 2004). HLT functions as a guide toward guides the design of instructional materials that have to be developed or adapted. In addition, HLT can be elaborated and refined while conducting the experiment.

Moreover, another prominent characteristic of design research is its cyclic character of which there are two kinds of cycles i.e. macro cycles and micro cycles (Bakker, 2004). Macro cycles comprehend of three phases namely design, teaching experiment, and retrospective analysis. Meanwhile, micro cycles relate to a set of problems and activities during one lesson. Considering the availability of the time for conducting the research, in this study, three consecutive cycles were administered of which the students participating in the first cycle was different to those who participated in the second cycle and in the third cycle. The retrospective analyses of a cycle lead to the refinement of the HLT of the next cycle.

The initial HLT was arranged in three activities. The first activity, which was in the form of hands-on activity i.e. providing stuff to hold by students, consisted of two problems adapted from the study of English (2007). The first one refers a two-dimensional problem (snacks and drinks): 2 kinds of snacks - 3 kinds of drinks, and 2 kinds of snacks - 4 kinds of drinks. The choice of the numbers of 2 snacks and 3 – 4 drinks were set so since they were considerably quite simple enough as a start. Meanwhile, the second problem broadens the dimension of the first problem becoming a three-dimensional problem (snacks, drinks, and fruits): 2 kinds of snacks - 3 kinds of drinks - 2 kinds of fruits and 2 kinds of snacks – 4 kinds of drinks - 2 kinds of fruits. They are aimed to lead the students using their strategies, mainly expected with odometer strategy (English, 2007), of which students make all possible combinations of one kind of snack and one kind of drink for the two dimensional problems and one kind of snack, one kind of drink, and one kind of fruit for the three dimensional problems.

The second activity was the extension of the previous activity although it was designed without hands-on activity which aimed at leading students to use multiplication operation in determining the number of all possible combinations. Besides that, it consisted of one two-dimensional problem and one three-dimensional problem. Specifically, the former one asks the students to determine the number of all possible combinations of the color of shirts and the color of trousers taken from four different shirts and five different trousers. Moreover, the latter one extends the former problem consisting of three different shirts, four different trousers, and four different hats. Here, the choice of the numbers of the objects are larger than those in the first activity to stimulate students to exhaustively count one by one and to use multiplication operation instead.

Furthermore, the students' comprehending of the multiplication principle is expected from the third activity which is in a form of hands-on activity. In more detail, the activity was aimed at making students aware of the similarity of two or more identical things when being combined with another object. The problem includes two kinds of snacks of which one of them consists of two identical things and two kinds of drinks.

The conjectures of students' thinking in this HLT were determined by adapting the works of students in the study of English (2007) and also thinking all possible ways the students can do with the problems. The following table describes the overview of the first cycle's HLT

Table 3.1. The First Cycle's HLT

Activity	Goal	Problem	Conjecture of students' thinking and learning
1	Students can list and determine the number of all	2 kinds of snacks - 3 kinds of drinks	• Some students will use trial-and-error approach.

Activity	Goal	Problem	Conjecture of students' thinking and learning
	possible two-dimensional pair combinations using odometer strategy	2 kinds of snacks - 4 kinds of drinks	• Other students will use cyclic pattern approach. • The other students will use odometer pattern approach. In determining the number of the combination total, the students rely on counting the object one by one
	Students can list and determine the number of all possible three-dimensional pair combinations using major-minor strategy	2 kinds of snacks - 3 kinds of drinks – 2 kinds of fruits 2 kinds of snacks – 4 kinds of drinks – 2	• Some students will use trial-and-error approach. • Other students will use major-minor strategy approach. In determining the number of the combinations, the students rely on

Activity	Goal	Problem	Conjecture of students' thinking and learning
		kinds of fruits	the records/note they make and count the objects one by one.
2	students can list and determine the number of possible two-dimensional combinations using multiplication.	4 different shirts – 5 different trousers	• Some students use trial-and-error method. • Other students will use cyclic pattern approach. • The other students will use odometer pattern approach. In determining the number of the combinations, the students rely on their records/note and count the objects one-by-one.
	students can list and determine the	3 different shirts – 3 different	• Some students will use trial-

Activity	Goal	Problem	Conjecture of students' thinking and learning
	number of possible three-dimensional combinations using multiplication.	trousers – 4 different hats	and-error approach. • Other students will use major-minor strategy approach.
3	Students can understand the concept of multiplication principle	2 kinds of snacks (one of them consisting of two identical things) and 2 kinds of drinks	Most students consider the two identical things are different each other when being combined with the drinks and the other consider it as two same things.

Four students of which the numbers of boys and girls are equal actively participated in the first cycle: Irwan, Tasya, and Gelya are 11 years old and Fauzan is 10 years old. Their schools are all located in Makassar, one of metropolitan cities in Indonesia. The rational of the subject choice and the ages are that they have already studied, at least memorizing, the multiplication 1 to 10. In addition, the choosing of the small number of the research subjects was aimed to study their activities and reasoning in depth. The students had not studied combinatorics. In addition, they were randomly selected from two state schools located in the middle class and one non-state school located in the downtown area. In each cycle, the researchers themselves acted as a teacher and the observers.

Data collection

In general, the data in this study were obtained from the preparation of the experiment and the experiment of all cycles. They were gathered by doing an interview, observing, and collecting written documentation. The interview and the observations were recorded by using field note and video to collect information e.g. the grade and the mathematics ability of students. Documents which were mainly collected in the experiment phases were student's written works.

Validity and Reliability

The issues of the validity and the reliability in this study mainly refer to the study of Miles and Huberman (1994) and the study of Bakker (2004) of which internal validity, external validity, internal reliability, and external reliability should be noticed. They are all concerned in qualitative way. Internal validity refers to the data collection quality and the considerable reasoning which can be used to draw conclusion. Then in this study, it was gained by collecting the different types of data (data triangulation) such as video recording, audio recording, photographs, field notes, and written work of the students. Different teaching experiments were conducted in all of the cycles aimed, one of them, to test the conjectures set in the earlier experiment in the later experiment. External validity or the generalizability is the extent to which one can generalize the findings from the contexts used in this study to other contexts which can be issued by presenting the findings of this study clearly so others can transfer it to their domains. Internal reliability means the extent to which the inference and the argumentation are reasonable. In this study, it was improved by discussing crucial activities with colleagues to minimize the sense of subjectivity and doing careful collection to the data e.g. coding the audio transcript and making video fragment. External reliability means replicability which has a criterion i.e. trackability of which a researcher should report the succession of his research in such a way that a reader can track his activities during research.

CHAPTER IV

RESULTS AND ANALYSES

As the nature of the design research which is cyclic character, this chapter covers the discussion of the first cycle as well as the its analysis for the preparation of the next HLTs. It also includes the analyses and the discussion of the second cycle's HLT and the third cycle's HLT.

A. The First Cycle Experiment and the Analysis

Activity 1

Irwan and Tasya are group-mate, say the first group, and Gelya and Fauzan are together in the second group. They all did the first two dimensional-problem in the first activity using trial and error approach. They were uncertain whether there are other ways to solve the problem. Both groups kept using the approach in solving the second problem. They didn't miss all of the possible combinations in both problems since they thoroughly grasped and matched each object of snacks with each object of drinks and wrote down the results one by one in the table available in the worksheet. Similar to the two dimensional-problem, both groups used trial and error approach to find all combinations of one snack, one drink, and one fruit in three dimensional-problem. Also, there was no possible combination which was missed. Since all of them had no idea of arranging all lists of snack-drink combination using odometer strategy, the researcher itself told them how to do it with that convenient way in two dimensional-problem. It was implemented aimed to lead them to multiplication concept in the next activity.

Activity 2

The second group used odometer strategy in solving the first problem and obtained 20 possible combinations. Meanwhile, the first group kept using trial and error way and arduously solved the problem with 19 possible combinations as the result with one missing couple. When Tasya and Irwan looked how the second group did the problem, they realized that its work was more efficient. The researcher then did an interview to the second group aimed to know whether they came up with the multiplication concept:

Researcher : How many possible combinations in total?
Gelya and Fauzan : twenty
Researcher : How do you come up with twenty?
Gelya : Because there are twenty couples in the table
Researcher : Exactly, how do you get all of the combinations?
Gelya : We match the white shirt first to all of the trousers then it was the same with yellow shirt, red shirt, and green shirt.
Researcher : How many matches for each shirt?
Gelya and Fauzan : five
Researcher : how many fives then?

Gelya : four, so five added by five three times, so the total is twenty

In the fragment, the second group knew the total by seeing the whole combination list in the table. It seemed that the use of multiplication was still subtle since they related it to the addition operation.

The researchers thought that the second problem in the second activity previously set in the HLT was quite unlikely for students to solve using multiplication since they didn't come up with the idea of multiplication in the previous problem and the numbers of the problem were quite high. Changing the problem become a simpler one was done to replace the initial problem. The numbers of shirts, trousers, and hats were reduced from respectively 3, 3, and 4 to 2 for each. This minor change during experiment is allowable (Bakker, 2004) when researchers have an objective in avoiding difficult activities. As a result, the first group listed the first four combinations by maintaining using the black shirt and matching it with red trouser firstly and blue trouser alternately and also green hat and yellow hat alternately. The next four combinations were done using the same method, however, the shirt kept by the first group was white. Meanwhile, the second group listed the first two combinations using major-minor strategy by keeping the black shirt as the major component and red trouser as the minor component. For the next two combinations, it assigned the blue trouser as the minor component. However, for the remains, the second group did the same as the first group did. The works of both groups are shown in figure 4.1. where *baju* is shirt, *celana* is trouser, and *topi* is hat.

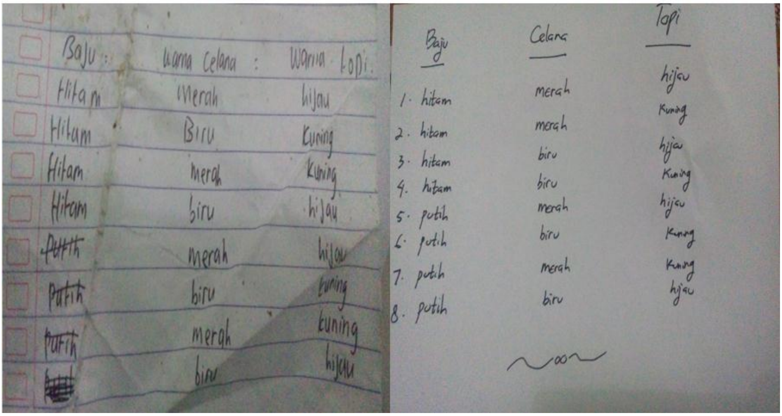


Figure 4.1. The work of the first group (left) and the work of the second group (right)

Then the researchers did an interview with the first group to explore their ideas.

Researcher : how many couples in total if there are two shirts, two trousers, and two hats?

Irwan : eight

Researcher : what about there are one shirt, two trousers, and two hats?

Tasya : four

Researcher : why is it four?

Tasya : because for black shirt, there are four couples

Researcher : what about there are three shirts, two trousers, and two hats?

Irwan : it will be twelve couples
Researcher : why?
Irwan : since everyone shirt addition will result the four pair accretion
Researcher : what about there are four shirts, two trousers, and two hats?
Tasya : it will be sixteen couples
Researcher : will it be the same when there are two shirts, four trousers, and two hats?
Tasya : it will simply the same
Researcher : what is your reason?
Tasya : since for each shirt there are four trousers then it will make in total eight couples for shirt and trouser. Next, each of the eight couple

Furthermore, the researchers applied a separated interview to the second group with similar questions previously asked to the first group. The answers and the argumentations of the second group were quite similar to those of the first group.

Based on the latest interview, it is interpreted that using major-minor approach quite help the students immediately count the number of possible combinations based on the number of the objects covered by one major component of the combinations. Moreover, the strategy can help both groups to have a comprehending of determining the number of possible combinations although the numbers of each object are altered.

Activity 3

In this activity, surprisingly, almost students in both groups considered the two combinations with the same kinds of objects were the same meaning in which they only counted them once except Irwan who counted it twice and had discussion in his group with Tasya and also the other group about that difference. They attempted to make Fauzan cross his mind that the two combinations were the same. Furthermore, to make Fauzan aware, the teacher described him flag analogy with two equal horizontal bands : red-white, i.e. there are two red bands and one white band which make one kind of flag. He then concluded that his answer was incorrect.

B. Second Cycle's HLT

As in the first cycle, specifically in the first activity when the teacher himself told directly the students how to work with the problem using efficient strategies, the RME's philosophy i.e. *inventing mathematics* was not perceived quite satisfying, the researchers discussed to make an improvement to the HLT. The argument of Eizenberg and Zaslavsky (2003) that simplifying the number of objects without changing the essence of a problem motivated the researchers to encourage the students to start with "small number". In detail, the kinds of snack were altered from two to one and it would be matched to respectively two and three kinds of snacks. Then, the next problem was related to the previous of which the number of snacks were increased from one to two and there were three kinds of snacks. It was rationalized that the use of one object urges the students could be accustomed to keep using an object to match with another kind of objects. Besides that, it simplifies the acquaintance of number patterns. To directly connect the concept of the multiplication stemming from the *odometer* strategy, it was decided that the two-dimensional problems without hands-on activity was set to replace the three-dimensional problems with hand-on activity as the continuance and made it as one of the problems in the second activity. The Eizenberg and Zaslavsky's argument was also used as the rationale of modifying that which was in the second activity in the first HLT. To make the use of the multiplication clear, the researchers added three consecutive problems, i.e. 2 different shirts – 2 different trousers, 2 different shirts – 3 different trousers, 2 different shirts – 4 different trousers. These additional problems were planned to ask to the students before asking the previously existed problem in the first HLT. The use of such kinds of problems was hypothesized to lead the students to identify the pattern of the numbers and connect it with the use of multiplication concept.

Furthermore, for the second activity, in the three-dimensional problems with hands-on activity comprehending two consecutive problems, the compositions set in the questions respectively were 1 snack – 2 different drinks – 3 different fruits and 2 different snacks – 2 different drinks – 3 different fruits. Meanwhile, for the there-dimensional problems without hands-on activity, it comprehended of 1 shirt – 2 different trousers –

3 different caps, 2 different shirts – 2 different trousers – 2 different caps, and 3 different shirts – 2 different trousers – 2 different caps. Since there was no difference between the actual and the hypothesized students' thinking, there is no significant change made to the activity 3, unless a plan to add some questions for an improvisation. The second cycle's HLT is described in the following table:

Table 4.1. The First Cycle's HLT

Activity	Goal	Problem	Conjecture of students' thinking and learning
1	Students can list and determine the number of all possible two-dimensional pair combinations using odometer pattern approach	1 kind of snack - 2 kinds of drinks	The students will get 2 possible combinations by pairing the snack to each of the drinks.
		1 kind of snack - 3 kinds of drinks	The students will get 3 possible combinations by pairing the snack to each of the drinks
		2 kinds of snacks – 3 kinds of drinks	• Some students will get 6 six possible combinations. They keep the combinations they get in the previous problem

Activity	Goal	Problem	Conjecture of students' thinking and learning
			and then pairing the other snack to the other drinks. Automatically they just make an addition in determining the number of the combinations. • The other students start pairing the snacks and the drinks from the beginning using trial and error strategy In determining the number of the combination, the students count the couples made one by one

Activity	Goal	Problem	Conjecture of students' thinking and learning
	students can list and determine the number of possible two-dimensional combinations using multiplication.	2 different shirts – 2 different trousers	All students will use *odometer* strategy. In determining the number of the combinations, the students answer 4. Even they firstly know that 4 is the answer by doing addition two plus two.
		2 different shirts – 3 different trousers	All students will use *odometer strategy*. In determining the number of the combinations, the students answer 6. Even they firstly know that 6 is the answer by doing addition four plus two

Activity	Goal	Problem	Conjecture of students' thinking and learning
		2 different shirts – 4 different trousers	All students will use *odometer strategy*. In determining the number of the combinations, the students answer 8. Even they firstly know that 8 is the answer by adding six by two
		4 different shirts – 5 different trousers	The students answer 20 by multiplying 4 by 5. They identify already the patterns and conclude that it uses multiplication.
2	Students can list and determine the number of all possible three-dimensional pair combinations	1 snack - 2 different drinks – 3 different fruits	• Some students will use *major-minor* strategy • The other students will use trial and error strategy

Activity	Goal	Problem	Conjecture of students' thinking and learning
	using major-minor strategy	2 different snacks - 2 different drinks – 3 different fruits	• Some students will use *major-minor* strategy. They simply continue the work from the previous problem. In determining the number of the combinations, they just make an addition. • The other students will use *trial and error* strategy. They count the combinations one by one to get the total.
	students can list and determine the number of possible three-	1 different shirts – 2 different trousers –	The students will use major-minor strategy. They multiply one (the number of shirt) by two (the number of

Activity	Goal	Problem	Conjecture of students' thinking and learning
	dimensional combinations using multiplication.	2 different hats	trousers) and next pairing the shirt-trouser combination to the hats and get 4.
		2 different shirts – 2 different trousers – 2 different hats	The students will use major-minor strategy and simply multiply 4 by 2
		3 different shirts – 2 different trousers – 2 different hats	The students will use major-minor strategy and simply multiply 8 by 2
		2 different shirts – 5 different trousers – 4 different hats	The students will multiply 2 by 5 by 4 to get 40
3	Students can understand the concept of	2 kinds of snacks (one of	Some students consider the two identical things are

Activity	Goal	Problem	Conjecture of students' thinking and learning
	multiplication principle	them consisting of two identical things) and 2 kinds of drinks	different each other when being combined with the drinks and the other consider it as two same things.

Six students which were in the same school consisting of three boys and three girls participated in the second cycle. They were divided into two groups, say the first group and the second group, consisting of three students for each. The academic abilities ranging from low to high are represented by these students who study in SD IBA which is in Palembang. In addition, in each group there were a high achiever, a middle achiever, and a low achiever. Heterogeneous gender was also identified in each group. They followed all the activities in the second cycle. The method of the data collection was the same as that of in the first cycle.

C. The Results and the Analysis of the Second Cycle's HLT

Activity 1

As being hypothesized, the students did the first and the second problem by pairing the snack with each of the drinks available. Specifically, both groups held the snack and moved it nearby or the top of each drink alternately, thus they got 2 and 3 as the answers respectively. Meanwhile for the third problem, they applied *odometer* strategy by firstly pairing one of the snacks to each of the drinks and doing the same for the other snacks. However, unpredictably, this activity is different to the set hypothesis which predicted that the students who used odometer strategy would simply continue pairing the snacks and the drinks from the activity in the second problem, instead, the students definitely started pairing and writing the combinations from the beginning. In determining the total of the combinations, they saw the total of the combinations resulted from their written works.

Moreover, when working without objects provided, i.e. the context of shirts and trousers, surprisingly, the students mentally answered the total of the combinations first before listing the distinct pairs of a shirt and a trouser. They trivially knew that multiplying the number of shirts and the number of trousers is the method to know the total of the combinations. Furthermore, when being given the last problem, the students knew that the answer was 20. When being asked aimed to guide them how they came up with the number of the combinations, they reflected on the solution patterns from the previous problems. The listing they made was just for assuring that the number of the combinations was the same as the number obtained by multiplication. Their conceptions were more firmly established when they were able to explain that the number could be obtained using addition since they saw from the strategy they used.

Activity 2

In the first problem, the second group applied the *major-minor* strategy. Interestingly, it wrote down the drinks first and completed with the snack and the fruits as the second and the third component respectively of which the last component was definitely the most frequently changed component. The writing of the combinations of the second group is shown in the figure 4.2.

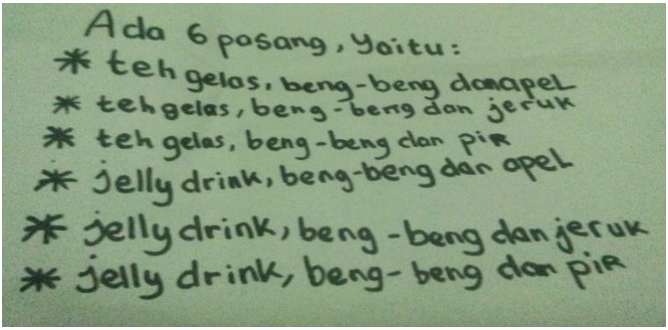


Figure 2. The work of the second group of the three-dimensional problem

On the other hand, the first group made the fruits as the minor component and the drinks as the most frequently changing items. Like in the two-dimensional problem, as shown in the figure 2. both groups knew the answer before listing the possible combinations one by one. To know how they come up with the answer, the researchers did an interview with the second group with some important fragments as follows:

<u>Rec 1</u>

Researcher : which one did you answer first? The six or you wrote down the combinations first and later you knew that there were six combinations?

Marvin : answering six and writing the combinations

Researcher : how did you predict that six?

Marvin : six (thinking)

Reza : three times two times one

Marvin : yes, that is

Reza : there was one snack, there were two kinds of drinks, and there were three fruits, so it was three times two times one.

......

The reason why they used multiplication is described in the following interview fragment rec 2 :

<u>Rec 2</u>

Researcher : why using multiplication?

Reza : since using the way like this (pointing out the combinations written in the paper work) is harder

Researcher : but why was it should be the multiplication?

Marvin : to get the result easily, it's faster

Researcher : but how do you know that it should be multiplication?

Reza : the problem that was given there were two and two (two snacks and two drinks) becoming four

Marvin : also there was one snack and three drinks, if it was added becoming four combinations, if being multiplied becoming three combinations, and the answer was three combinations.

Based on the latest fragment, the second group knows that the multiplication was used by reflecting on the results of the previous problems. The consistency of the answer pattern that suits to the multiplication of the numbers of each item led the students to use the multiplication to know the number of possible combinations. In addition, after being interviewed regarding to the answer, the use of multiplication to get six was also applied by the first group. However, the researchers felt difficulty to explore their ideas since the students simply explained that multiplying the numbers of each item was a simple and a quick way to get the answer. In addition, they perceived that the strategy used by the second group was efficient in listing the combinations since it would cause less change when the drinks became the *minor* component

Moreover, when the number of kinds of snacks was altered becoming two, unlike the previous problem, the first group was not able to directly answer the total number of the possible combinations, instead, it established the combinations one by one first by using *major-minor* strategy. Specifically, its work was similar to how the second group did the latest problem by forming a pattern of which the students held and wrote the first kind of drinks constantly for the first three combinations while keep maintaining the first kind of snack. These three drinks and snacks then was completed with a fruit which was different each other. Later the students continued the pattern with the second kind of drink for the next three combinations whose pattern like the previous three combinations. After completing and seeing the whole possible combinations, the students then saw that the total was 12. On the other hand, the second group showed a significant progress by simply multiplying six derived from the possible combinations in the previous problem by two since the second snack also caused the other six combinations. It also asserted that multiplication of each item numbers was used for this kind of problem i.e. $2 \times 3 \times 2$. Similarly, the first group, it was capable of using the *major-minor* strategy for writing every combination for this problem.

Activity 3

Firstly, the students in both groups undoubtfully determined that the total combination was six. When they all were asked why it was six, they applied *odomoter* strategy in pairing each snack available to each drink without holding the things in front of them. They had an argument that the two *Betters*, i.e. the kind of snack consisting two identical things would result in different combinations when each of them was paired with a drink. Then the teacher holding the two *Betters* and *Teh Gelas*, i.e. one of the drinks, promoted discussion by asking them whether they were different combinations. By seeing the combinations of the snacks and the drink hold by the teacher, the students then were aware of their incorrect conception and considered that the two combinations were simply the same.

After that, the teacher made an improvisation to pose a question to students by increasing the number of *Better* to four and the number of *Teh Gelas* becoming two, all of them consistently answered that the total combinations were still as many as four.

HLT

D. The Third Cycle's HLT

The activities designed in the second cylce's HLT made the students systematically list the entire combinations of objects. However, what they did was just simply listing the combination one by one using numbering or bullet which is named listing-odometer method which was assumed as the cause of students not grasp the multiplication principle concept as shown in the figure 4.2.

Consequently, they used multiplication because of the inductive reasoning they applied reflecting from the results of some problems. Several mathematics discrete textbooks in which the multiplication principle is covered and pedagogic literatures in teaching multiplication e.g. (Fosnot & Dolk, 2001) inspired the researchers to introduce a multiplication model i.e. tree diagram model to evoke students to come up with the multiplication concept. The set HLT consists of two activities. The first activity related to the 2D problems of which two goals are set, i.e. firstly, students can list and determine the number of all possible two-dimensional pair combinations using odometer pattern approach and secondly, by being skillful in using such approach, the students can grasp the multiplication concept and use it to solve some more complex problems. To obtain the first goal, the activity was set based on the statement of Eizenberg and Zaslavsky (2003) that simplifying the number of objects without changing the essence of a problem is one of the solutions to help students in learning combinatorics. Specifically, the number of an item should be set as least as possible. In this HLT, a sequence of 1-2, 1-3, and 2-3 were addressed to the number of kind of snack – the number of kind of drinks. It was expected to students that after they make a listing of 1-3 snack and drinks, i.e. pair the snack to each of the drink, they simply continue to the other snack to pair to each of the drink in solving the 2-3 problem. Moreover, the snack-drink part involves a hands-on activity of which students use physical object incorporated to the learning (Lineberger & Zajicek, 2000) as the researchers reflect on its effectivity for students to encompass all of the combinations in the previous cycles. There is also 2-2 attributed for the number of two different shirts-the number of two different trousers for the next problem which is without hands-on activity. The conjectures of students' thinking were suggested by the answers of the students in English (2007), Höveler (2014), Yuen (2008), and the first's and the second's HLT. Tree diagram model is introduced in this stage as a respond to listing method answer.

Table 4.2. The First Cycle's HLT

Activity	Goal	Problem	Conjecture of students' thinking and learning
1	Students can list and determine the number of all possible two-dimensional pair combinations using odometer pattern approach	1 kind of snack - 2 kinds of drinks	The students will get 2 possible combinations by pairing the snack to each of the drinks.
		1 kind of snack - 3 kinds of drinks	The students will get 3 possible combinations by pairing the snack to each of the drinks
		2 kinds of snacks – 3 kinds of snacks	• Some students will get six possible combinations. They keep the combinations they get in the previous problem and then pairing the other snack to the other drinks other drinks

Activity	Goal	Problem	Conjecture of students' thinking and learning
			(odometer-listing method). They just make an addition in determining the number of the combinations. • The other students start pairing the snacks and the drinks from the beginning using trial and error strategy. In determining the number of the combination, the students count the couples made one by one

Activity	Goal	Problem	Conjecture of students' thinking and learning
	Teacher shows the comparison which strategy between the *trial and error* or the *odometer-listing* better. Next, the teacher introduces tree diagram model in bridging the conception of students from listing method to multiplication concept. Tree diagram model is used to solve the above problem		
	students can list and determine the number of possible two-dimensional combinations using multiplication.	2 different shirts – 2 different trousers	Some students will use *odometer* strategy with tree diagram model. In determining the number of the combinations, the students answer 4 by counting the combination one by one The other students still use listing method and get 4 as the answer by counting the combination one by one

Activity	Goal	Problem	Conjecture of students' thinking and learning
		Teacher shows students to compare which method more effective to encourage them in using tree diagram model.	
		2 different shirts – 3 different trousers	Using their inductive reasoning, most of the students have the assumption that the number of the combination can be obtained by multiplying the number of shirts and the number of trousers. They answer the number of the combination, i.e. 6, first before listing the combinations of the objects. Most students use tree diagram

Activity	Goal	Problem	Conjecture of students' thinking and learning
			model in listing the combinations.
		Teacher asks the students how many combinations without listing the combination of 5 different shirts – 3 different trousers	Some of them answer 15 by their inductive reasoning Some of them answer 15 since for each shirt can be paired to three trousers and since there are five shirts, there are fifteen combinations

Twelve 10-12 year old students divided into four groups participated in the experiment of which each group consisted of three students. They were studying in SD Athirah, an elementary school lying in downtown area of Makassar, namely one of crowdly populated area in Indonesia. The number of the students in this cycle was determined larger than that of in the second cycle consisting of six students aimed to obtain more comprehensive data. They were taken as samples by random purposive sampling technique who are heterogeneous in the term of mathematics ability. High, middle, and less mathematics ability could be found in every group. In additiion, each group was set unisex since, based on the discussion with their home-room teacher, it would simplified the students work cooperatively to their group mates if the students worked with the students with the same sex. Furthermore, the students haven't studied multiplication principle.

Following the first activity, the HLT for the second activity was set also based on the theory of (Le Calvez francoise.le-calvez@lip6.fr, Giroire helene.giroire@lip6.fr, & Tisseau gerard.tisseau@lip6.fr, 2008). It starts from 1-2-3 addressing the number of kind of snack-the number of kinds of drinks-the number of kinds of fruits.

2	Students can list and determine the number of all possible three-dimensional pair combinations using major-minor strategy	1 snack - 2 different drinks – 3 different fruits	• Some students will use major-minor strategy with tree diagram model • Some students will use major-minor strategy with listing method • The other students will use trial and error strategy In determining the number of the

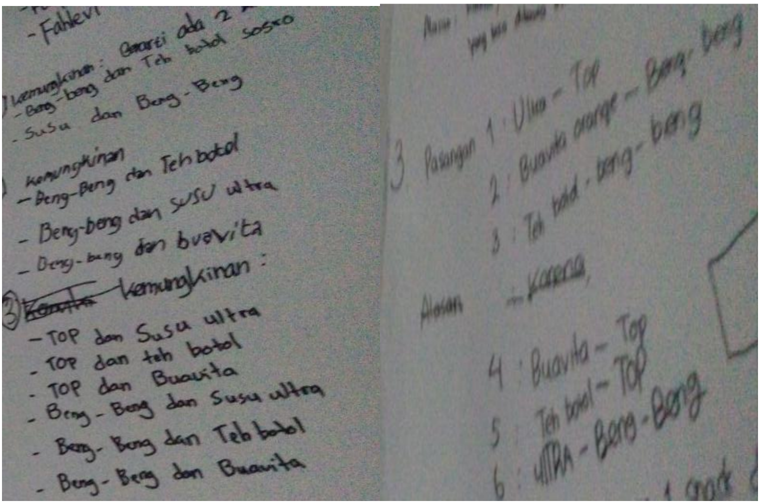
			combinations, most students count the combination one by one.
		Teacher let each group present its answer in the whiteboard and ask them to compare which answer simpler and more effective. In this case, teacher explains more the answer and uses the tree diagram model aiming to bridge the problem to the multiplication concept. Teacher firstly "group" and multiplies the major and the minor component as for the single major component there are some minor components and then for each group, there are some other components	
		2 different snacks - 2 different drinks – 3 different fruits	• Most students will use major-minor strategy with the tree diagram model. They simply continue the work

			from the previous problem. • The other students will use trial and error strategy. They count the combinations one by one to get the total. In determining the number of the combinations, some students just make an addition or counting one by one. Meanwhile, less students grasp the concept and apply multiplication, i.e. $2 \times 2 \times 3$

The First Activity

As the conjecture suggests, all students paired the snack to all of the drinks availabe for both 1-2 and 1-3 problem in the hands-on activity. However, some groups previously had considered that there were only one possibility of snack-drink a child can bring from the problem 1-2. Only after the teacher asked them whether the other pairs possible, the students understand that the total is not one but two instead No combination was missing also for 2-3 problem by the students.. In addition, The answers of the students for the problem were variative as the hypothesis suggests. Some groups applied *odometer-listing* strategy and the others use that of *trial and error*. The groups that understand the problem well starting from the first problem tended to use *odometer-listing* strategy. In determining the number of the combination, all groups did a counting

Figure 4.1. The works of the students: with odometer approach (left) and trial and error method (right)



Based on the guide from the HLT, the teacher showed the comparison of the two strategies they used and introduced them tree diagram model for solving the latest problem. In this step, the teacher hasn't yet introduced the concept of multiplication.

Furthermore, in the problem 2-2, it was found the variety of strategies the students used. Some groups used the tree diagram model and the other used odometer-listing method as the hypothesis indicates. All of them keep using counting to identify the number of the combination. After that, based on the guide, the teacher explained the addition concept lying in the answer using tree diagram model of which for each shirt, it can be paired to two trousers, so there are four in total. Then the teacher posed new problem, i.e. five different shirts and three different trousers without the colors and most students answered fifteen. To know the reason behind the answer, the teacher made conversation with one of the groups, namely as shown in the following recorded conversation fragment:

Teacher : why is it fifteen?

Student : because five times three

Teacher : why do you multiply five by three?

Student : since each shirt can be paired to three trousers and there are five shirts so it means five time three

The Second activity

Understanding a mathematics problem in the form of word often make students difficult to grasp the meaning of it. As in the two-dimensional problem, there were several students didn't understand well the problem implying unexpected answers, most of the groups didn't understand the following more complicated given 1-2-3- problem:

Izza is provided by her mother one kind of snack, i.e. *beng-beng*, two kinds of fruits, i.e. *teh botol sosro* and *susu ultra*, and three kinds of fruits, i.e. apple, orange, and banana. How many kinds of combination and what are the combinations when Izza simply want to have one snack, one fruit, and one fruit?

The teacher then explained what the problem was so it could be understandable for them by giving them more translation for the problem (Jupri & Drijvers, 2016). The explanation by the teacher made the students more confident to solve the problem. There were sort of different process in obtaining the list of the combination answer, i.e. major-minor-listing method, tree-diagram model, and trial and error. The teacher then let the group which used tree-diagram model and that which used major-minor-listing method to present their answers in the whiteboard aiming to use it to lead the students using multiplication. The group which used tree diagram model made the snack, i.e. beng-beng as the major part and the fruits as the minor part. That group which used major-minor-listing method also made beng-beng as the major part, however, and the drinks as the minor part. No missing combination found in all the groups' work and determined the number of the combination by counting.

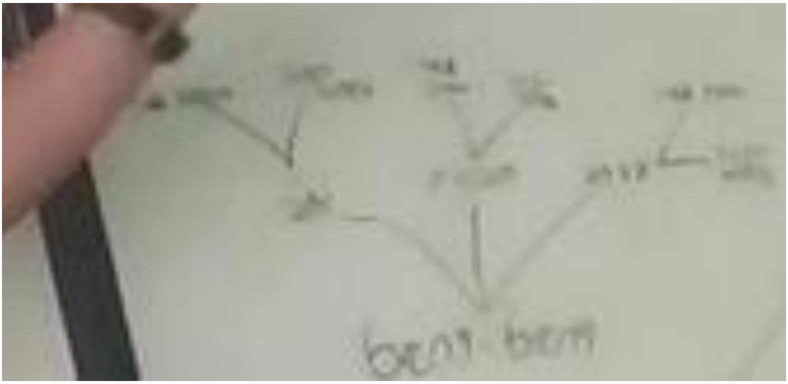


Figure 4.2. The work of students with tree-diagram model

The teacher then made a tree-diagram model, in contrast to the work of the group, and set the drinks as the minor part. The teacher asked the students whether the number of the was also six and then the students considered that it was also six by counting. The teacher then used the second problem in the HLT, 2-2-3 problem. Most students used tree-diagram model and counted ony by one the combination to obtain the total.
Till the last problem, there was no indication that the students grasped the concept of multiplication in the three-dimensional problems.

Most of the literatures evoking students to take the advantage of model and apply mathematics concept aiming not to use an exhaustive process like counting, tend to make the problems more complicated, e.g. increasing the number of objects teacher ((Gravemeijer & van Eerde, 2009), (Wijaya, 2008)). The teacher then decided to add the problem of which the number of drinks alter becoming three. Before the students worked with the model, the teacher initiated to ask the groups whether they know already the number of the combination. Most of them already realized the number although they had not created the tree-diagram model for the context. They could imagine from the tree-diagram model they set from the previous problem, i.e. 2-2-3 problem. The teacher observed one of the groups and asked them resulting to discussion as recorded and transcribed in the following fragment:

Teacher : how many combination in total?
Students : eighteen
Teacher : how do you know that it is eighteen?
Students : (pointing the already made tree-diagram model, exactly the minor part of the previous problem) it will be three. So this is three, three, three, three and (pointing the latest part of the tree diagram model since there was an addition one minor part, i.e. from two to three) this is three, three, three, and three
Teacher : so, what is the process in obtaining eighteen?
Students : (counting) one, two, three, then being added and so on until eighteen

Counting one by one method was also made by the other groups. Later on, in making the last attempt, since the limited time alloted for learning, to lead the students come up with the multiplication concept, the teacher made a whole discussion. Specifically, the teacher put emphasis on the number of minor part each major part has and the number of the last part each major-minor part have and the relationship among the problems related to multiplication. Starting from reexplaining the answer of 1-2-3 problem, i.e. the answer is six, the teacher then asked the students that how many combination if the number of snack becoming two. The students answered twelve since the new snack corresponded also to the six combination of drinks-fruits.

The researchers assumed that, if the problem was developed by altering the number of the snack becoming three, then the students would simply did binary operation, i.e. three times six which was considered that it would not lead the students to the concept of tertiary multiplication as there were three numbers in three dimensional problem. Then, the teacher decided to increase the number of drinks becoming three, i.e. 2-3-3. Most students then skillfully answered by using tree-diagram model of which the snack and the drink were the major and the minor component and, however, counted the combination one by one to get eighteen. Next, the teacher asked them to use another method in determining the number of the combination. The student who answered using odometer strategy from the beginning and proficiently used multiplication for the two dimensional problems nicely answered using multiplication for the latest problem. His explanation to the teacher and the other students was recorded in the following fragment:

Student : it is eighteen

Teacher : why is it eighteen?

Student : since it is six (pointing out the number of snacks and drinks)

Teacher : how do you get six?

Student : two times three

Teacher : why is it two multiplied by three?

Student : because one snack is paired to three drinks and there are two snacks, there are six combinations

Teacher : then go on

Student : these six pairs are paired to three fruits, so six multiplied by three equals eighteen

The student who explained the present answer didn't take the advantage the tree-diagram model available in the whiteboard by the previous student, instead, relying on the number of objects written by the teacher in the whiteboard.

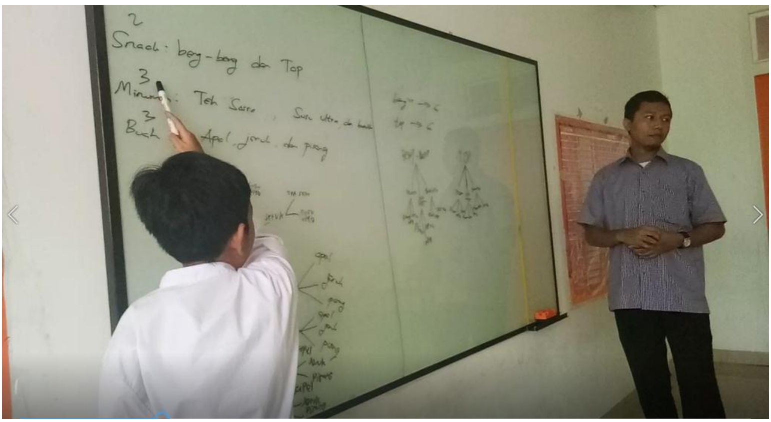


Figure 4.3. Student Work With Multiplication

CHAPTER V

CONCLUSION

This study was initially challenged with the question that how can the designed learning activities support elementary school children to apply efficient strategies in solving problem as well as to reach the understanding of multiplication principle concept?. The hands-on activities for all cycles in the beginning assist the students cover the whole combination of objects. The compositions of consecutive snack-drink lead them to apply listing-odometer strategy. The introduction of odometer strategy in the form of tree-diagram by the teacher influences the students choice of representing the combination of objects. Particularly, the students in the third cycle all eventually prefer the tree-diagram model because of its simplicity for large number of objects. The tree-diagram model in two-dimensional context simplifies several students to see how many objects can be paired to each object and then connect it to the multiplication concept instead of counting the combination one by one. Furthermore, some students who skillfully used the tree-diagram model from the beginning retain using the model in solving three dimensional problems. However, the model doesn't help the students grasp multiplication concept unless they are guided by the explanation of the teacher about how many objects the major-minor component has.

It should be noted that, although the students, based on an interview said that they mainly learned multiplication by memorizing, most of them see that multiplication as repeated addition. That understanding plays important role of the concept grasping in the learning. Although the goals of the learning are reached, the teacher focuses only on the students who follow the learning trajectory and tend to reach the learning goals smoothly and, based on the information from the school official, have high mathematics ability. The HLT simply tends to influence the other students by showing them the comparison of their answers and the sophisticated answers by their friends. It is suggested for further research to highlight the students having lack of mathematics abilities to guide them in grasping the desired mathematics concept.

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