LEARNING FROM MISCONCEPTION TO RE-EDUCATE STUDENTS IN SOLVING PROBLEMS OF MATHEMATICS

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Abstract

This study aims to recognize students' misconception and to re-educate students to correct mathematical thinking. By observational study, some mathematical topics were examined and information was collected to secondary graders to find factors influence students working in learning activities. There are four factors as fundamental aspect to observe misconception, such as faults result from organized strategies and rules, faulty rules underlying errors have reasonable origins, students perceive arithmetic as an activity isolated from their ordinary apprehensions, and students often display a crack between prescribed and familiar acquaintance. Those factors were surveyed using a number of problems in which trigger mistakes made by students within some examples in their real works.

Keywords: Learning, Misconception, Problems, Mathematics

1. Introduction

In line with the development mathematics learning in the classroom, more problems are being talked and more misconception are found. According to Malcolm Swan (2001), a 'misconception' is not wrong thinking but is a concept in embryo or a local generalization that the pupil has made. It may in fact be a natural stage of development. In any case, the misconception seems no problem in solving process. However, there is a counterexample that contradicts to the misconception such that it is against the belief system of thinking system.

Although we can and should steer clear of activities and examples that might encourage them, misconceptions cannot simply be avoided (Swan 2001: 150). In the other words, a teacher who wants to teach mathematical concepts particularly may be trapped in the misconception. So, he/she is not addressing students to help understanding the concepts well and precisely, instead of creating new trouble. Consequently, the system of thinking that is fundamental to build system of belief is built by the components of misconceptions.

There are many things that student think in mathematics such as, rules, importance, boredom, and enjoyment. They are part of their attitudes and thinking about mathematics. One problem that leads to very serious learning difficulties in mathematics is those misconceptions student may have from previous inadequate teaching, informal thinking, or poor remembrance. It may be best to begin with a definition. From the Encarta online dictionary, a misconception is "a mistaken idea or view resulting from a misunderstanding of something." While Pines (1985) stated that certain conceptual relations that are acquired may be inappropriate within a certain context. Here the relation is called "misconceptions." A misconception does not exist independently, but is contingent upon a certain existing conceptual framework. Misconceptions can change or disappear with the framework changes.

As professional instructor of mathematics learning, we should concern about the importance of misconception as the challenge or the obstacle to boost students' ability. But, it cannot be escaped that misconception would be "big stone" for teachers in which the process of transfer learning can be achieved imprecisely and ineffectively.

Based on the experience of classroom observation, there are two things that the facilitators of mathematics lesson should know. The first thing is how depth they understand about misconception, and how many kinds of misconception in mathematics learning they have recognized.

Getting started from those two concerns, this article exhibits some facts taken from classroom research and focus on how misconception appears as problems or obstacles for students in learning math. Also, the explanation of examples could be well experience to share better information about misconception in mathematical thinking.

2. Content and Method

Students come to the classroom as plain slates. In mathematics classes, research shows that students can enter the classroom holding misconceptions that have the strong potential to derail new learning (Brown, 1992; Chiu & Liu, 2004; Kendeou & van den Broek, 2005). This has enormous implications for classroom instruction. The presence of student misconception suggests teachers need to identify and target misconceptions and build up accurate conceptual knowledge all while still providing students with enough instruction and practice on the wealth of procedural skill that are required course components and likely targets of standardized testing. Researchers in the domains of cognitive development and cognitive science have identified an instructional technique which may be especially helpful in fitting all these needs: the use of worked example with self-explanation prompts.

There are some considerations related with why misconception can be happened to students on their thinking process, such as (1) translational errors (Clement, 1982), (2) poorly understand the reasons making, (3) careless to find the domain of answer, (4) misbelief of the algorithm, (5) not working well, (6) poor skilled at fundamental facts, (7) no idea of the execution plan, (8) abandon the rules or misinterpret in many types of simplification problems, (9) having trouble with the correct definition, (10) having trouble with precedence of operations, (11) misusing the distributive rule, (12) poor understand the difference between two related concepts and more, (13) improper understanding of.

Probably useful finding that revealed the most important findings of mathematics education research carried out in Britain over the last twenty years has been that all pupils constantly 'invent' rules to explain the patterns they see around them (Askew and Wiliam, 1995). While many of these invented rules are correct, they may only apply in a limited domain. When pupils systematically use incorrect rules, or use correct rules beyond their proper domain of application, we have a misconception. For example, many pupils learn early on that a short way to multiply by ten is to 'add a zero'. But what happens to this rule, and to a child's understanding, when s/he is required multiply fractions and decimals by ten? Askew and Wiliam note that It seems that to teach in a way that avoid pupils creating any misconceptions is not possible, and that we have to accept that pupils will make some generalizations that are not correct and many of these misconceptions will remain hidden unless the teacher makes specific efforts to uncover them.

Therefore it is important to have strategies for remedying as well as for avoiding misconceptions. Some strategies for avoiding and for remedying these misconceptions are then suggested as the matter of reeducating in the process of learning mathematics.

Talking about misconception, it is related with cognitive ability that it has been researched by many experts. According to Piaget, all cognitive change can be classified as one of two types: adaptation and organization. Organizationis a largely internal process involving rearranging and linking up items of previous learning to form a “strongly interconnected cognitive system” (Berk 1997: 213). More important for our purposes are adaptation, which itself comes in two varieties: assimilation and accommodation. In assimilation the learner simply fits new concepts, skills and information into his or her existing cognitive framework. However, on some occasions new items of learning cannot be fitted into the existing cognitive framework, and that framework must be changed in order to make room for them. This is accommodation.

The awareness of a need for a change in one’s cognitive framework is brought about by a realization that something important ‘doesn’t fit in’. For this reason, Malcolm Swan and others in the Diagnostic Teaching Project have seen Piaget’s views as providing theoretical justification for their view that the best way to overcome a misconception is by engineering a cognitive conflict (Swan, 2001).

Addressing misconceptions during teaching does actually improve achievement and long-term retention of mathematical skills and concepts. Drawing attention to a misconception before giving the examples was less effective than letting the pupils fall into the ‘trap’ and then having the discussion. (Askew & Wiliam, 1995: 13)

Students tend to be emotionally and intellectually attached to their misconceptions, partly because they have actively constructed them and partly because they give ready methodologies for solving various problems. They definitely interfere with learning when students use them to interpret new experiences.

It is very important to recognize student misconceptions and to re-educate students to correct mathematical thinking. Although the results apply more to children younger than high school age, Ginsberg (1977)offers a number of observations about errors: 1. Errors result from organized strategies and rules, 2. Faulty rules underlying errors have sensible origins, 3. Too often children see arithmetic as an activity isolated from their ordinary concerns. 4. Children often demonstrate a gap between formal and informal knowledge.

In particular, the last point on formal vs. informal knowledge requires definition. Usually, formal knowledge refers to that which is taught in an organized, structured, educational institution. It refers to a system of interrelated definitions and proofs, experiments and arguments. It usually is linked with written methods. On the other hand, informal knowledge refers to more tentative intuitive conjectures and mental strategies. Informal knowledge is generated or learned through one’s personal actions. That is, informal knowledge refers to routines that are carried out mechanically, or by habit, or by tradition.

According to online Merriam Webster, reeducation is raining to develop new behaviors (as attitudes or habits) to replace others that are considered undesirable. An understanding of common student misconception, and effective strategies to help students avoid them, is an important aspect of mathematical pedagogical content knowledge (Graeber, 1999). In addition to trying to teach in such a way that students avoid misconception must also have approaches for dealing with those that inevitably arise. Therefore, the target of reeducation here is developing new attitudes or perspective about mathematics to tend working math contextually. Then, the problem that it would foster is how such teacher to know in depth what kinds of misconception in mathematics learning they have recognized.

By observational study, some mathematical topics were examined and information was collected to secondary graders to find factors influence students working in learning activities. There are four factors as fundamental aspect to observe misconception, such as faults result from organized strategies and rules, faulty rules underlying errors have reasonable origins, students perceive arithmetic as an activity isolated from their ordinary apprehensions, and students often display a crack between prescribed and familiar acquaintance. From this, there are some evidences of misconception in which we as teachers can take benefit to encourage students following their learning process in the right way. Those findings are discussed in the next result and discussion.

3. Result and Discussion

There are four concerns about misconception as learning for the result of observation, which are errors result from organized strategies and rules, faulty rules underlying errors have sensible origins, too often students see arithmetic as an activity isolated from their ordinary concerns, students often demonstrate a gap between formal and informal knowledge. In order to understand those, since the meaning of misconception should underline the main thinking before making conclusion that it is misconception, each of concerns are completed with examples, such as students' answers.

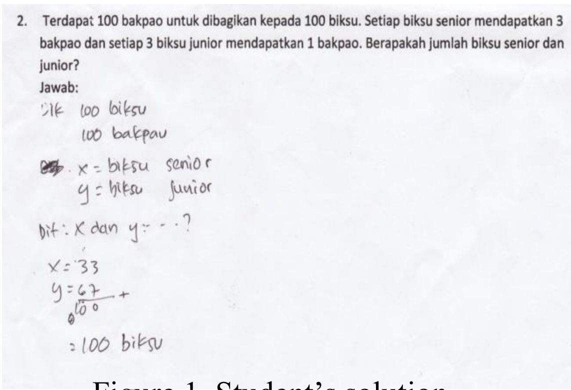
2. Terdapat 100 bakpao untuk dibagikan kepada 100 biksu. Setiap biksu senior mendapatkan 3 bakpao dan setiap 3 biksu junior mendapatkan 1 bakpao. Berapakah jumlah biksu senior dan junior?
Jawab:
Dik 100 biksu
100 bakpau
x = biksu senior
y = biksu junior
Dit: x dan y = ...?
x = 33
y = 67 +
100
= 100 biksu

Figure 1. Student's solution

Firstly, errors result from organized strategies and rules, meaning that the error of result that students show off in their solution caused by inappropriate strategies and imprecise rules. As evidence, below is one of student's solutions about mathematical problem.

Looking at figure 1, we can see ordered steps which are divided into three parts. Firstly, identification problem step, the student write "Dik 100 biksu, 100 bakpau. Subsequently, there two variables using to represent biksu senior and biksu junior are x and y, respectively. Before going to execute the plan, the questions are sentenced to what is x and y? However, it seems not easy to understand that x = 33 and y = 67, the total is 100 biksu. Shortly, it reminds us how the way of problem solving can be useful to arrange their idea. But, how come is to get 33 and 67? This becomes irrationally since there is no appropriate reason to support that the number of biksu senior is 33, and the number of biksu junior is 67.

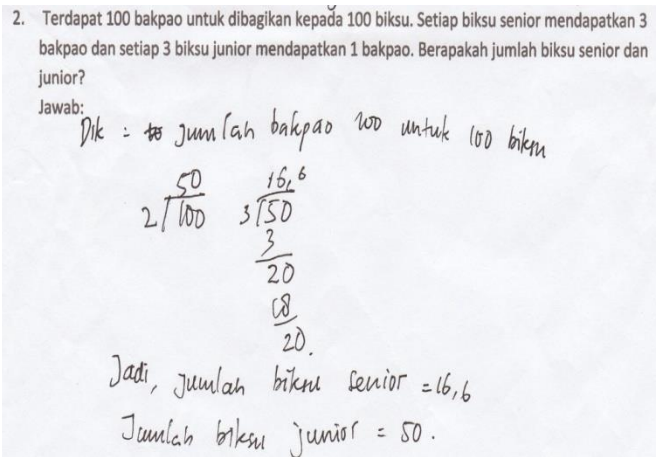

Figure 2. Student's solution

Compared with figure 2, it is not quite different with figure 1. In the other words, to answer the problem student also arranges problem solving steps which may be clearer than the first one. It means that this answer show us how to find out the number of biksu senior by dividing 50 with 3, then it is 16,6. Before determining why 50, the student also divided 100 by 2, so the number of biksu junior is 50. The problem then is how can we interpret biksu senior 0,6, and the total between 16,6 and 50 does not reach 100.

Based on these two worked examples, it seems that not only strategies they applied underlining the errors, but also the plan is improper to determine the solution. From this, teacher should stop teaching calculation, start teaching mathematics (Wolfram, 2010)

Secondly, faulty rules underlying errors have sensible origins, this kind of misconception actually is the indication to show that adding function is not always easy for some students in topic relation and function. The next problem given to the students is addition of two functions. If $f(x) = 2x - 1$ and $g(x) = x^2$, then what is the result of $(f+g)(x^2)$?

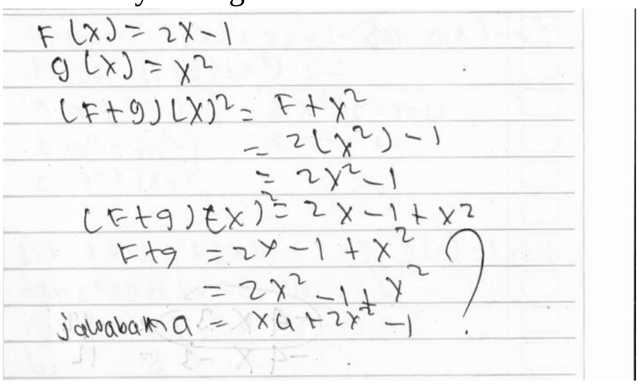

Figure 3. Student's solution on adding two functions

The main idea of this mathematical problem is how student determine the result by addition, arithmetic concept, but it is related with function. Actually, the awareness of students that variable is also open sentence which can be changed into numbers is not reached when they consider it different with. The answer in figure 3 exhibit us the knowledge about what we should insert into the variable x on function. This problem make the student confused about "$(f + g)(x^2)$", since he substituted "$(f + g)(x^2)$" equal with $f + x^2$, following this $2(x^2) - 1$ or $2x^2 - 1$. This is challenging teacher to make reform of thinking that substitution is not composition, even though we can continue to substitute the variable after composite the functions. More important for students who are learning functions that either $f(x)$ or $g(x)$ is function of x. So, if $f(x^2)$ or $g(x^3)$, then f is function of x^2 and g is function of x^3.

That's why that it becomes ambiguous to understand supported arguments of the final answer, $x^4 + 2x^2 - 1$. Started from $(f + g)(x^2) = 2x - 1 + x^2$, it should be $(f + g)(x^2) = 2(x^2) - 1 + (x^2)^2 = 2x^2 - 1 + x^4$. Although the answer is $x^4 + 2x^2 - 1$, then he write in his answer sheet. But, logically it is not enough supported reason to accept it. In this case, the solution consists faulty rules to support mathematical thinking in solving the problem. Therefore, the final answer does not come from strong arguments, even though it is correct based on the answer key.

Thirdly, too often students see arithmetic as an activity isolated from their ordinary concerns. This leads students to think the problems as a challenge remote from their origins. By harnessing their knowledge about numbers, especially the properties of the traditional operations between them — addition, subtraction, multiplication and division; students elaborate their arithmetic knowledge to do calculating without concerning logical reasoning related the problems. Clearly, figure 2 is the example how the student construct their idea without considering selected operation to execute some facts to find the solution.

Related to the figure, probably we become confused to understand 100 divided by 2. The result of interview to the student who explains about his strategy stated that because there are two groups, biksu senior and biksu junior. He got 50, then it is divided by 3 since every biksu shared 3 bakpao each other. The next trouble come to this answer, fractional part of 16,6. It becomes irrational when it is deal with the number of people.

Lastly, students often demonstrate a gap between formal and informal knowledge. Formal knowledge refers to that, which is taught in an organized, structured, educational institution. It refers to a system of interrelated definitions and proofs, experiments, and arguments. It usually is linked with written methods. On the other hand, informal knowledge refers to more tentative intuitive conjectures and mental strategies. Informal knowledge is generated or learned through one's personal actions. That is, informal knowledge refers to regular activities that are carried out instinctively, or by tendency, or by custom.

In order to find out the information that students comprehend about ratio, proportion, and fraction, the question is asked leads to excavate explanation of those. There are two explanations about ratio that we have to consider with.

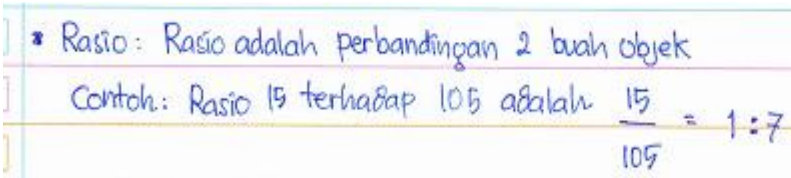

Figure 4. Student's explanation of ratio

Using layman's terms a ratio represents, for every amount of one thing, how much there is of another thing. a ratio is a relationship between two numbers of the same kind (e.g., objects, persons, students, spoonfuls, units of whatever identical dimension), expressed as "*a* to *b*" or *a:b*, sometimes expressed arithmetically as a dimensionless quotient of the two that explicitly indicates how many times the first number contains the second (not necessarily an integer).

Compared with the definition of student above, ratio is comparison between two objects. This definition does not compatible with its example. Although the definition is in general which probably is interpreted the object as the quantity, it means that either object or quantity is the thing. However, "rasio 15 terhadap 105" is one of forms to represent ratio, while another is *15/105*. It is quite different when the ratio of *15/105* is also *1:7*, because the last ratio is another ratio, even though it is proportional with *15/105*. For example, supposing one has 15 oranges and 105 lemons in a bowl of fruit, the ratio of oranges to lemons would be *1:7* (which is equivalent to *15:105*) while the ratio of lemons to oranges would be *7:1*. Additionally, but the number of oranges of 15 pieces in a bowl is different with that only 1 piece in a bowl. So, ratio cannot be assumed as division, even though mathematical expression of ratio can be written like the expression of division.

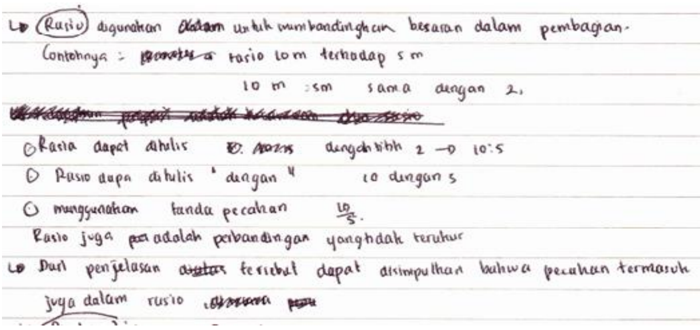
Rasio digunakan untuk membandingkan besaran dalam pembagian.
Contohnya : rasio 10 m terhadap 5 m
10 m : 5m sama dengan 2,
○ Rasio dapat ditulis dengan titik 2 → 10 : 5
○ Rasio dapa ditulis ' dengan " 10 dengan 5
○ menggunakan tanda pecahan 10/5.
Rasio juga adalah perbandingan yang tidak terukur
Dari penjelasan tersebut dapat disimpulkan bahwa pecahan termasuk juga dalam rasio

Figure 5. Student's explanation about ratio

Like an example that student revealed in his answer (figure 5), "contohnya: rasio 10 m terhadap 5 m, 10m : 5m sama dengan 2"; rasio dapat ditulis dengan titik 2 → 10 : 5". This is likely doing operation of division between 10 and 5. Actually, this condition is what we called a gap between formal definition and informal knowledge. Students sometimes need constructing formal definition of mathematical knowledge in terms of informal knowledge in order to accommodate and assimilate the knowledge by their understanding.

The four factors as fundamental aspect to observe misconception are completed with the examples that we found in observational study in the classroom research. This scientific activity brings us to do more research, especially in belief system. Like Swan (2001), a 'misconception' can be not wrong thinking. However, it can be no longer to be true when it fosters to counterexample related to the problem. By the discussing this aspect which boost misconception to students in learning process, we or who else should consider the factor that encourage students to defend their misunderstanding about anything, for example mathematical concepts.

4. Conclusion

Misconception is not something that teacher should be afraid of, but it becomes the challenge. In the classroom, the teachers need to make their students understand by constructing their idea. For long lasting process, students are engaged with knowledge that is probably not to be true. That's why students are trained to develop their logical reasoning to filter whatever information they receive.

Using knowledge of misconception and some examples related with in which teacher and students are getting experience is the true effort in order to reeducate either teacher as facilitator learning process or students are being active to enjoy the process of learning mathematics.

The four factors that have been identified, which are faults result from organized strategies and rules, faulty rules underlying errors have reasonable origins, students perceive arithmetic as an activity isolated from their ordinary apprehensions, and students often display a crack between prescribed and familiar acquaintance. Following the aspects are the examples gotten from observational study, and the explanation is to discuss what the cause of misconception is.

Using the example of students' misconception as learning material is in order to reeducation process in which teacher and students are engaged with the useful experience to reach better goals of learning.

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