PAPER • OPEN ACCESS

Mathematical Modeling of APKA on Dynamics Unemployment Rate in Makassar City

To cite this article: A Ansar *et al* 2021 *J. Phys.: Conf. Ser.* **1899** 012089

Mathematical Modeling of APKA on Dynamics Unemployment Rate in Makassar City

A Ansar[1)], R Syam[2)], S Sidjara[2)] and S Side[2,*)]

[1] Student of Magister Department, Universitas Negeri Makassar, Indonesia
[2] Department of Mathematics, Universitas Negeri Makassar, Indonesia

***Email**: syafruddin@unm.ac.id

Abstract. This research explain about mathematical modeling APKA type on dynamics unemployment rate in makassar city. The aim of this research is how to to analyze and simulate the model to predict the enemployment rare in makassar city which composed is two cases, the first is case of the unemployment free rate and the second is case increase unemployment rate. The type of the research is an applied research which of aims to solve the problem with compiling consept as needed. The data used are secondary data form Badan Pusat Statistik (BPS) in makassar city, by using basic reproduction number (R_0) to determine the dynamics unmemployment rate in makassar city. If $R_0 \leq 1$ then the condition of the number of unemployment population is not worrying, but $R_0 > 1$ decribes the condition increasing the number of unemployment population which need to be followed up by goverment of makassar city. The simulation result found $R_0 = 0{,}8445 \leq 1$, that means, the number of unemployment population is not worrying and the other simulation found $R_0 = 1{,}8718 > 1$, that means, the number of unemployment population is worrying in Makassar city. The result also shown that if the number of individual not in the worksforce is hinger then the number of unemployed will be hinger.

1. INTRODUCTION

Unemployment is a measure that a person takes if who people is not have a job but they have been actively trying to find job in the last four weeks [1]. The problem of unemployment have many negative impact, one of which is descrease income and economic growth, increase destitution and intensify of crime. In south sulawesi, particularly makassar city occupies the top position in terms of unemployment which 10.39% of the total population [2]. Unemployment can be overcome by increasing investment, incresing quality of human resources, expand an employment and set up a traning center.

Researches on social cases have been explain about the problems of economic growth and unemployment in a social perspective [3-5]. But they have not considered it in the mathematical aspect. The researches about mathematical modeling SIR, SIRS, SEIR and SEIRS have been carry out on [6-21], which model are applied not only caseses of dengue fever, tuberculosis, hepatitis B, diphteri, measles but also applied to social issues among others like drugs, online game addiction and corruption. Those researches have not applied the SIRS model to social problems in the case of unemployment rate problem.

This research discuss about SIRS modeling on enemployment problems but most of the SIRS models are applied in spread of disease. But in this research discuss about social problems so that the reseachers forming mathematical modeling of APKA then perform with analysis and simulation to secondary data from Badan Pusat Satistik (BPS) makassar city to predict the growth rate of unemployment in makassar city

2. METHOD

This research use an applied method which aim to solve the problem and compile the concepts to the needs. The mathematical model is the suspect, infected, recovered and suspect [12], which is modified into mathematical model of APKA. The analysis of the APKA model for unemployment using the lynapunov and routh hurwitz method [13,19]. The simulation of the APKA model using Maple and the data of this model simulation is secondary data of unemployment in Makassar city from BPS Makassar.

3. RESULT

3.1. Formulation of Model

Diagram of population changes in the unemployment rate problem with APKA model in figure 1:

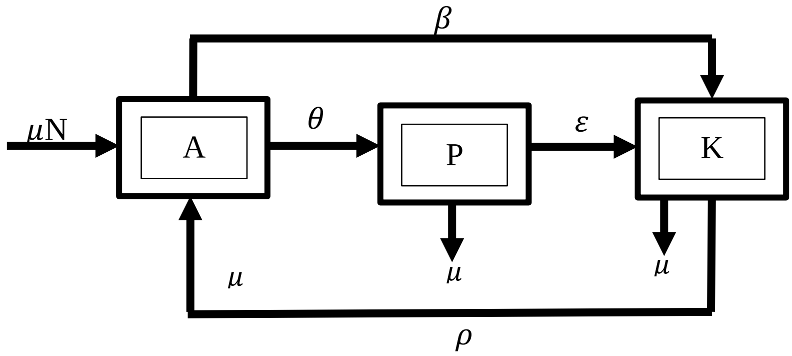

Figure 1. Diagram model of APKA

The total population can be seperate in three classes: the first is *not the workforce* (A) states the number of individuals not in the labor force, second is *open enemployment* (P) states the number of individuals open unemployment and the last is *working* (K) states the number of individual who have found the job. The individual in the class *working* (K) can move back to *not the workforce* (A) and the individual in not *the workforce* (A) can go straight to *working* (K) class.

Table 1. Definition and parameters in model APKA

Parameters	Definition
N	Number of individual workforce and not workforce
θ	Rate of movement population individual class of *not the workforce* (A) to population individual class of *open unemployment* (P)
ε	Rate of movement population individual class of *open enemployment* (P) to population individual class of *working* (K)
ρ	Rate of movement population individual class of *working* (K) back to population individual class of *not workforce* (A)
β	Rate of movement population individual class of *not the workforce* (A) to population individual class of *working* (K)
μ	Total of population are over 15 years of age and individual who experience death or retirement.

Accorrding to figure 1 has obtained mathematical model of APKA for the unmemployment rate is as in equations (1)-(3):

$$\frac{dA}{dt} = \mu N + \rho K - (\beta + \theta + \mu)A \tag{1}$$

$$\frac{dP}{dt} = \theta A - (\varepsilon + \mu)P \tag{2}$$

$$\frac{dK}{dt} = \varepsilon P + \beta A - (\rho + \mu)K \tag{3}$$

3.2. Analysis of APKA Model for unemployment

3.2.1. Equilibrium point for matematical model APKA of unemployment free rate.
Unemployment free rate assumed there is no unemployed human population. To determine of the equilibrium point for the unemployment free rate obtained by making the left side in equation (1) until (3) become to zero. Futhermore equilibrium point of unemployment occur if $i = 0$. So as the equlibrium point of unemployment is:

$$(E_0) = \left(\frac{\mu N(\rho+\mu)}{(\rho+\mu)(\beta+\theta+\mu)-\rho\beta)}, 0, \frac{\beta\mu N}{(\rho+\mu)(\beta+\theta+\mu)-\rho\beta}\right)$$

3.2.2. Equilibrium point for unemployment rate increase.
Determination of equilibrium point of unemployment rate increase obtained by making the left side in equation (1) until (3) become to zero. Futhermore looking for the solution of variables values of A, P and K so as equilibrium point for unemployment rate increase is:

$$E_e = \left(\frac{\mu N(\varepsilon+\mu)(\rho+\mu)}{(\varepsilon+\mu)\big((\rho+\mu)(\beta+\theta+\mu)-(\rho\varepsilon\theta+\rho(\varepsilon+\mu))\big)}, \frac{\theta\mu N(\rho+\mu)}{(\varepsilon+\mu)\big((\rho+\mu)(\beta+\theta+\mu)-(\rho\varepsilon\theta+\rho(\varepsilon+\mu))\big)}, \frac{\mu N\,(\varepsilon\theta+\beta(\varepsilon+\mu))}{(\varepsilon+\mu)\big((\rho+\mu)(\beta+\theta+\mu)-(\rho\varepsilon\theta+\rho\beta(\varepsilon+\mu))\big)}\right) \tag{4}$$

3.2.3. Determination of basic reproduction number
Basic reproduction number (R_0) determine using of *next generation matrix* method from equation (2) in mathematical modeling of APKA then we have:

$$\varphi = \theta A \text{ and } \omega = (\varepsilon + \mu)P \tag{5}$$

Furthermore, linearization of θA and $(\varepsilon + \mu)P$ and we obtained,

$$F = \theta \text{ and } V = (\varepsilon + \mu)P \tag{6}$$

Subsequently searched is V^{-1} and we have:

$$V^{-1} = \frac{1}{\varepsilon+\mu} \tag{7}$$

Next generation matrix obtained by multiply F and V^{-1} so that:

$$R_0 = K = FV^{-1} = \theta\left(\frac{1}{(\varepsilon+\mu)}\right) = \frac{\theta}{\varepsilon+\mu} \tag{8}$$

3.3. Basic reproduction number analysis of APKA model for unemployment

3.3.1. Analysis of unemployment free rate

Theorem 1.
If $R_0 \leq 1$, then equilibrium of unemployment free rate P^* has stable global asymptote.

Proof.
Suppose the candidate for lyapunov function on the equation (7) is:

$$\mathcal{L}(t) = (A - A^* \, Ln\, A) + P + K \quad (9)$$

By differentiating equation (9) with time we obtained equation (10):

$$\begin{aligned}
\mathcal{L}'(t) &= A^*\left(1-\frac{A^*}{A}\right) + P^* + K^* \\
&= (\mu N + \rho K - ((\beta+\theta+\mu)A))\left(1-\frac{A^*}{A}\right) + \theta A - (\varepsilon+\mu)P + \varepsilon P + \beta A - (\rho+\mu)K \\
&= \mu N\left(1-\frac{A^*}{A}\right) - (\beta+\theta)A\left(1-\frac{A^*}{A}\right) - \mu A\left(1-\frac{A^*}{A}\right) + \theta A + \beta A - \mu P - \rho K - \mu K + \rho K\left(1-\frac{A^*}{A}\right) \\
&= \mu N\left(1-\frac{A^*}{A}\right) + \mu A^*\left(1-\frac{A^*}{A}\right) + (\beta+\theta)A^* - \rho K\frac{A^*}{A} - \mu P - \mu K \\
&= \mu N\left(1+1-\frac{A^*}{A}-\frac{A}{A^*}\right) - \left(\frac{\rho K}{A} - (\beta+\theta)\right)A^* - \mu P - \mu K \\
&= \mu N\left(2-\frac{A^*}{A}-\frac{A}{A^*}\right) - \mu P - \mu K \\
&= -\mu N\left(\frac{(A-A^*)^2}{AA^*}\right) - \mu P - \mu K \quad (10)
\end{aligned}$$

Because $\mathcal{L}'(t) < 0$ then equilibrium of unemployment free rate has stable global asymptotic.■

3.3.2. Analysis of the increasing unemployment rate.

The next step is to analyze the stability of the previously obtained equlibrium point. Stable here is defined as a condition where if the equilibrium point is disturbed, it will return in an equilibrium state and to analyze the stability of the equlibrium point, the first is determine the jacobian matrix from equation (1)-(3):

Let Jacobian matrix E_e in the following equation:

$$JE_{\mathrm{e}} = \begin{bmatrix} -(\beta+\theta+\mu) & 0 & \rho \\ \theta & -(\varepsilon+\mu) & 0 \\ \beta & \varepsilon & -(\rho+\mu) \end{bmatrix} \quad (11)$$

Based on jacobian matrix in equation (11) then the charachteristic equation is: $\det(\lambda I - JE_{\mathrm{e}}) = 0$

$$\det\left(\lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -(\beta+\theta+\mu) & 0 & \rho \\ \theta & -(\varepsilon+\mu) & 0 \\ \beta & \varepsilon & -(\rho+\mu) \end{bmatrix}\right) = 0$$

$$\det\begin{pmatrix} \lambda+\beta+\theta+\mu & 0 & -\rho \\ -\theta & \lambda+\varepsilon+\mu & 0 \\ -\beta & -\varepsilon & \lambda+\rho+\mu \end{pmatrix} = 0$$

and we have the charachteristic equation in (12) as follow:

$$\lambda^3 + \lambda^2(3\mu+\varepsilon+\beta+\theta+\rho) + \lambda(\beta\varepsilon+2\beta\mu+\theta\varepsilon+2\theta\mu+2\mu\varepsilon+\varepsilon\rho+2\mu\rho+\theta\rho++3\mu^2) + \beta\varepsilon\mu+\varepsilon\mu\theta+\mu^2\theta+\theta\rho\mu+\mu^2\rho+2\varepsilon\rho\mu+\beta\mu^2+\mu^3+\mu^2\varepsilon = 0 \quad (12)$$

Then the characteristic equation is obtained in equation (13).

$$k(\lambda) = \lambda^3 + L_1\lambda^2 + L_2\lambda + L_3 \quad (13)$$

The Huwitz stability test is used to determine the type of eigenvalues of the matrix, i.e though determinat of matrix *H* which called Hurwitz matrix. In general form the matrix *H* is a matrix with entries as in equation (11). So that sufficent and necessary condition $k(\lambda)$ has an eigenvalues with negative real number as follow:

$$H = \begin{bmatrix} L_1 & 1 & 0 \\ L_3 & L_2 & L_1 \\ 0 & 0 & L_3 \end{bmatrix} > 0$$

a. If $[L_1] > 0$

$L_1 > 0$

b. $\begin{vmatrix} L_1 & 1 \\ L_3 & L_2 \end{vmatrix} > 0,$

$L_1L_2 - L_3 > 0$

$2\varepsilon\beta\mu + \varepsilon^2\beta + \beta^2\varepsilon + \theta\beta\varepsilon + \rho\beta\varepsilon + 6\mu^2\beta + 2\varepsilon\beta\mu + 2\beta^2\mu + 2\theta\beta\mu + 2\beta\rho\mu + 3\mu\varepsilon\theta + \theta\varepsilon^2 + \theta\varepsilon\beta + \theta^2\varepsilon + \theta\varepsilon\rho + 6\mu^2\theta + 2\theta\mu\varepsilon + 2\theta\mu\beta + 2\theta^2\mu + 2\theta\mu\rho + 6\mu^2\varepsilon + 2\mu\varepsilon^2 + 2\mu\varepsilon\beta + \mu\varepsilon\theta + 2\mu\varepsilon\rho + \mu\rho\varepsilon + \varepsilon^2\rho + \varepsilon\rho\beta + \varepsilon\theta\rho + \varepsilon\rho^2 + 6\mu^2\rho + 2\mu\rho\varepsilon + 2\mu\rho\beta + \mu\rho\theta + 2\mu\rho^2 + 3\mu\theta\rho + \varepsilon\theta\rho + \theta\rho\beta + \theta^2\rho + \theta\rho^2 + 8\mu^3 + 2\mu^2\varepsilon + 2\mu^2\beta + 2\mu^2\theta + 2\mu^2\rho > 0$

$L_1L_2 > L_3$

$3\varepsilon\beta\mu + \varepsilon^2\beta + \beta^2\varepsilon + \theta\beta\varepsilon + \rho\beta\varepsilon + 6\mu^2\beta + 2\varepsilon\beta\mu + 2\beta^2\mu + 2\theta\beta\mu + 2\beta\rho\mu + 3\mu\varepsilon\theta + \theta\varepsilon^2 + \theta\varepsilon\beta + \theta^2\varepsilon + \theta\varepsilon\rho + 6\mu^2\theta + 2\theta\mu\varepsilon + 2\theta\mu\beta + 2\theta^2\mu + 2\theta\mu\rho + 6\mu^2\varepsilon + 2\mu\varepsilon^2 + 2\mu\varepsilon\beta + 2\mu\varepsilon\theta + 2\mu\varepsilon\rho + 3\mu\rho\varepsilon + \varepsilon^2\rho + \varepsilon\rho\beta + \varepsilon\theta\rho + \varepsilon\rho^2 + 6\mu^2\rho + 2\mu\rho\varepsilon + 2\mu\rho\beta + 2\mu\rho\theta + 2\mu\rho^2 + 3\mu\theta\rho + \varepsilon\theta\rho + \theta\rho\beta + \theta^2\rho + \theta\rho^2 + 9\mu^3 + 3\mu^2\varepsilon + 3\mu^2\beta + 3\mu^2\theta + 3\mu^2\rho > \beta\varepsilon\mu + \varepsilon\mu\theta + \mu^2\theta + \theta\rho\mu + \mu^2\rho + 2\varepsilon\rho\mu + \beta\mu^2 + \mu^3 + \mu^2\varepsilon$

c. $\begin{vmatrix} L_1 & 1 & 0 \\ L_3 & L_2 & L_1 \\ 0 & 0 & L_3 \end{vmatrix} \begin{vmatrix} L_1 & 1 \\ L_3 & L_2 \\ 0 & 0 \end{vmatrix} > 0$

$L_1L_2L_3 - L_3^2$

$L_3(L_1L_2 - L_3) > 0$ or positive value

Based on above condition in the equation (13) we have negative eigenvalue so the conclusion is equilibrium point is stable based on Hurwitz criteria.■

3.4. Simulation of APKA Model for Unemployment

Mathematical simulation are carried out to give an overview about numeric simulation of mathematical modeling on unemployment rate in makassar city. The simulation is carried out by software and subtituting the initial values obtained based on research data and some assumed of parameters.

Table 2. Variables in model of unemployment dynamic

Variables	Value of Variable	Source
A	$\frac{\text{the number of individual not the worksforce}}{\text{the number of population}} = \frac{456.989}{1.128.033} = 0{,}405$	BPS,2019
P	$\frac{\text{the number of individual open unemployment}}{\text{the number of population}} = \frac{81.823}{1.128.033} = 0{,}072$	BPS,2019
K	$\frac{\text{the number of individual working}}{\text{the number of population}} = \frac{589.221}{1.128.033} = 0{,}522$	BPS,2019

Table 3. Parameter $R_0 \leq 1$ in model of unemployment dynamic

Parameters	Values of parameters	Source

	$R_0 \leq 1$	
θ	0,288	Assumption
β	0,583	BPS, 2019
ε	0,338	BPS, 2019
ρ	0,005	PHI-JamSos, 2018
μ	0,003	Assumption
N	1.128.033	BPS, 2019

Table 4. Parameter $R_0 > 1$ in model of unemployment dynamic

Parameters	Values of parameters	Source
	$R_0 > 1$	
θ	0,672	Assumption
β	0,583	BPS,2019
ε	0,338	BPS,2019
ρ	0,005	PHI-JamSos, 2018
μ	0,021	Assumption
N	1.128.033	BPS, 2019

3.4.1. For $\boldsymbol{R_0 \leq 1}$

Subtitution the parameters values in table 3 to equations (1)-(3) and obtained:

$$\frac{dA}{dt} = 0{,}003 + 0{,}005K - 0{,}874A$$
$$\frac{dP}{dt} = 0{,}288A - 0{,}341P$$
$$\frac{dK}{dt} = 0{,}338P + 0{,}583A - 0{,}008K$$

Futhermore, the subtitution of the parameters value in table 3 to the equlibrium point of unemployment free rate:

Table 5. Equlibrium values when $R_0 \leq 1$

Compartment	Value
A^*	0.009057736429
P^*	0.007649935752
K^*	0.9832923278

By using maple software, the eigen values λ are obtained as follows:

$$\lambda_1 = -0{,}0030000000$$
$$\lambda_2 = -0{,}8763099698$$
$$\lambda_3 = -0{,}3436900302$$

Since the values of λ are all negative values then this type of stability at this equilibrium point is stable.

Furthermore substituting parameter values in Table 3 to equation (8), found the basic reproduction number value of $R_0 = 0{,}8445 \leq 1$. This show that based on mathematical modeling of APKA then unemployment rate in makassar city is not worrying state, so that the individual enemployment will descrease along with the time and the number of individual working will increase. Simulation model can be shown in figure 2.

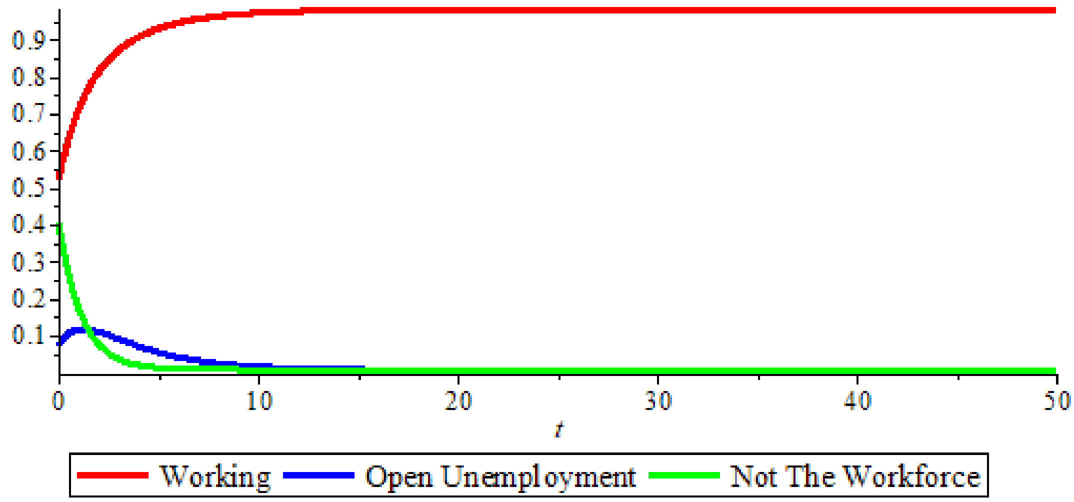

Figure 2. The curve of dynamic modeling of APKA unemployment rate ($R_0 \leq 1$)

The number of individuals not the workforce around 182.796, the individual have decreased significant, then they will in stability state after fifth years as 10.218 individuals.

The number of open unemployment individuals increased in the first three years from 6.546 individuals to 9.819 individuals. Subsequently the decreased drastically until the twentieth and the situation began to balance after the thentieth years as 8.630 individuals.

The number of individual working increased from 353.533 individuals to more than 571.545 individuals and they will in stability after tenth years as 1.109.187 individuals.

3.4.2. For $\boldsymbol{R_0 > 1}$

Subtititution parameters values in table 4 to equations (1)-(3) and obtained:

$$\frac{dA}{dt} = 0{,}021 + 0{,}005K - 1{,}276A$$
$$\frac{dP}{dt} = 0{,}672A - 0{,}359P$$
$$\frac{dK}{dt} = 0{,}338P + 0{,}583A - 0{,}026K$$

Futhermore, the subtitution of the parameters value in table 4 to the equlibrium point of unemployment free rate:

Table 6. **.** Equlibrium values when $R_0 > 1$

Compartemen	Value
A^*	0.02014942611
P^*	0.03771703160
K^*	0.9421335423

By using maple software, the eigen values λ are obtained as follows:

$$\lambda_1 = -0{,}02100000000$$
$$\lambda_2 = -1{,}277341229$$
$$\lambda_3 = -0{,}3626587707$$

Since the values of λ obtained are all negative then this type of stability at this equilibrium point is stable.

Futhermore subtituting parameter values in table 4 to equation (7) so we have the value of $R_0 = 1{,}8718 > 1$. This show that based on mathematical modeling of APKA then unemployment rate in makassar city is worrying state, so that the goverment of makassar city needs to follow up on the problem of unemployment. Simulation model can be shown in figure 3.

Figure 3. The curve of dynamic modeling of APKA unemployment rate ($R_0 > 1$)

The number of individuals not the workforce around 137.097, the individual have decreased significant, then they will in stability state after three years as 22.730 individuals.

The number of open unemployment individuals increased from 7.365 individuals to 15.547 individuals. Subsequently the decreased drastically until the twentieth and the situation began to balance after the thentieth years as 42.547 individuals.

The number of individual working increased from 353.533 individuals to 547.976 individuals and they will in stability after tenth years as 1.1062.758 individuals.

4. DISCUSSION

Research on matemtahical modeling of SIRS in cases of disease spread has been carried out by mulyanah [8], which will analyze the transmission of influenza in virus infected population with different number of initially infected individuals. Subsequent research by side [14] discusses building a model for dengue hemorrhagic fever, determine the stability of equilibrium point by using the lyapunov function, determine the value of the basic reproduction number, simulate model and intrepretation it.

Research on SIRS modeling in social cases shows that drug users are devided into two types, the first is who are not in the treatment period and the second is who are in the treatment period [6]. Research on mathematical modeling of SIR in social corruption cases discusses social problems, that is corruption is modeled in mathematics by making the corruotion behavior handled by komisi pemberantasan korupsi (KPK) modeled into a corruption control model to study stability and forms of control stategies that are effective to reducing the number of corruption actors in indonesia [16]. Research on the issue of unemployment rates shows that the couse of unemployment is few a job, the goverments effors to provide training are still nat a maximal and a lazy culture that still affects the job seekers [5].

This research discusses about the social problems of unemployment which are modeled in matematical modeling of APKA based on the SIRS model. This research aims to predict the growth rate of unemployment in makassar city.

5. CONCLUSION

Analysis of the APKA model explains the stability and equilibrium point on rate of unemployment problem and simulation result using software show that the result of analysis of the APKA model of unemployment in makassar city for unemployment taht takes into basic reproduction number ($R_0 > 1$) obtained the three population of group have a stabilities condition which means that the number of each population group had not changed in certain period of time. The number of not the worksforce population in a state of stabilities on 28.730 individuals. The number of open unemployment population in a state of stabilities on 42.547 individuals and the number of working population in a state of stabilities on 1.062.758 individual.

REFERENCES

[1] Sukirno, S. 1994. *Pengantar Teori Ekonomi*. Jakarta: Raja Grafindo Persada.

[2] Badan Pusat Statistika Kota Makassar Buku *Kota Makassar dalam Angka Tahun 2019.* https://makassarkota.bps.go.id/subject/6/tenaga-kerja. [accessed,16 Sept 2019].

[3] Alghofari, F. 2010. Analisis Tingkat Pengangguran di Indonesia Tahun 1980-2007. *Skripsi*.Universitas Diponegoro. Semarang.

[4] Burhanuddin. 2015. Pertumbuhan Ekonomi Tak Berkualitas dan Perlunya Kebijakan Pemerintah dalam Perluasan Kesempatan Kerja di Kota Makassar. *Jurnal Administrasi Publik.* Vol. 5, No. 2. Hal. 94-105.

[5] Franita, R. 2016. Analisa Pengangguran di Indonesia. *Nusantara (Jurnal Ilmu Pengetahuan Sosial)*. Vol. 1 No. 1. Desember 2016. Hal. 88-93. ISSN: 2541-657X.

[6] Ariesy, D. E. 2018. Pemodelan Penyebaran Pengguna Narkoba. *Jurnal Matematika UNAND*. Vol VII (3): 21-26.

[7] Aziziah, A.N., 2017. Model SIR Pada Epidemi Penyakit Campak Berdasarkan Umur dengan Pengaruh Vaksinasi. MATH-unesa *Jurnal ilmiah Matematika*. Vol. 3, No. 4.

[8] Mulyanah, L. 2008. Model SIRS pada Proses Penularan Penyakit Influenza dengan Populasi yang Terinfeksi Virus. *Skripsi* Institut Pertanian Bogor.

[9] Muzakir, N. A. 2019. Solusi Permasalahan Sosial Kecanduan Game Online Menggunakan Pemodelan Matematika SEIR Pada Siswa di SMP Negeri 3 Makassar. *Skripsi* Universitas Negeri Makassar.

[10] Rosdiana. 2015. Pemodelan Matematika SIR dengan Vaksinasi ada Penyebaran Penyakit Hepatitis B (Studi Kasus Provinsi Sulawesi Selatan). *Skripsi* Universitas Negeri Makassar.

[11] Said, C. S. 2019. Model SEIRS Penyebaran Penyakit Tuberkulosis di Kota Makassar. *Skripsi.* Universitas Negeri Makassar

[12] Side, S., Alimuddin, Bani, A. 2018. Modifikasi Model SIR pada Penyebaran Penyakit Demam Dengue di Kabupaten Bone. *Journal of Mathematics, Computations, and Statistics.* Vol. 1. No. 2, 169-182.

[13] Side, S., Zaki, A., Sari, N. 2018. Analisis Model Matematika Penyebaran Demam Berdarah Dengue dengan Fungsi Lyapunov. *Journal of Mathematics, Computations, and Statistics*. Vol.1. No. 2, 125-141.

[14] Sinuhaji, F. 2015. Model Epidemi Sirs Dengan Time Delay. Visipena. Vol. 6 No. 1. Januari-Juni 2015. Hal. 79-80. ISSN: 2086-1397.

[15] Wardiman, Kasbawati, Kusuma. J. 2018. Analisis Kestabilan dan Kontrol Optimal Model Matematika Perilaku Korupsi. *Skripsi.* Departemen Matematika. Universitas Hasanuddin.

[16] Wulandari, U. N. 2013. Analisis Model Epidemik MSEIR Pada Penyebaran Penyakit Difteri. *Skripsi*. Jawa Timur: Universitas Jember.

[17] Nur, W., Rachman, H., Abdal, N.M., Abdy, M., Side, S. 2018. SIR Model Analysis for Transmission of Dengue Fever Disease with Climate Factors Using Lyapunov Function. *Journal of Physics*. Vol. 1028 (1).

[18] Rangkuti, Y.M., Sinaga, M.S., Marpaung, F., Side, S. 2014. A VSEIR model for transmission of tuberculosis (TB) disease in North Sumatera, Indonesia. *AIP Conference Proceedings*. Vol. 1635: 201-208.

[19] Side, S., Mulbar, U., Sidjara, S., Sanusi, W. 2017. A SEIR model for transmission of tuberculosis. *AIP Conference Proceedings*. Vol. 1830.

[20] Poerwanto, B., Fa'Rifah, R.Y., Sanusi, W., Side, S. 2018. A matlab code to compute prediction of survival trends in patients with DHF. *Journal of Physics*. Vol. 1028 (1).

[21] Side, S., Irwan, Mulbar, U., Sanusi, W. 2017. SEIR model simulation for Hepatitis B. *AIP Conference Proceedings*. Vol. 1885.