Volume Unit in the Period of The Prophet Muhammad *Sallallahu Alayhi Wa Sallam*: An Integrated Study of Mathematics and Islamic Histories within the Contemporary Context

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ABSTRACT.

This article aims at presenting one of the mathematics topics in Islamic history, namely volume unit which had been used by Arabians in the medieval age, especially by the Prophet Muhammad *Sallallahu Alayhi Wa Sallam*. It is something rarely revealed by, or even never be found in the literature of mathematics history by Western mathematics historians. The study results show that in the period of Muhammad *Sallallahu Alayhi Wa Sallam*, Arabians used *Qullah* unit in determining the volume of an object, where 1 *Qullah* is about 96 liters so that 2 *Qullah*s will be about 192 liters. It is in line with the result of the computation mathematically based upon the explanation of scholars in contemporary Islamic Jurisprudence.

Keywords: Volume Unit . Arabian . Mathematics History

INTRODUCTION

One of the most important factors promoting the advance of mathematical science under Islamic civilization since the Prophet Muhammad *Sallallahu Alayhi Wa Sallam* is the attempt of Muslims in implementing Islamic laws and undertaking their worships as perfect as possible to get the blessing of Allah. There are several argumentations of the Qur'an and the Hadith which motivate Muslims to advance sciences and to apply them in practicing and doing their religious and living matters (Basya, 2015).

One of the Hadiths related to the implementation of Islamic laws and the development of mathematical science is the hadith about the volume of water that is sufficient to fill 2 *Qullahs* (pitchers). The Prophet *Sallallahu Alayhi Wa Sallam* has once said,

وَعَنْ عَبْدِ اللهِ بْنِ عُمَرَ رَضِيَ اللهُ عَنْهُمَا قَالَ: قَالَ رَسُوْلُ اللهِ صَلَّى الله عَلَيْهِ وَسَلَّمَ: إِذَا كَانَ المَاءُ قُلَّتَيْنِ لَمْ يَحْمِلِ الخَبَثَ وَفِيْ لَفْظٍ:

لَمْ يَنْجُسْ (أَخْرَجَهُ الأَرْبَعَةُ وَصَحَّحَهُ ابْنُ خُزَيْمَةَ وَالْحَاكِمُ وابْنُ حِبَّانَ)

"If the water is more than 2 Qullahs, it will not become filthy." Another narration says, "It does not become impure." (Related by the four imams: Abu Dawood, Nasa'i, Tirmidhi, and Ibnu Majah. Ibnu Khuzaimah, Al-Hakim, and Ibnu Hibban graded it as Sahih). On the other Hadith concerning purification *(taharah)*, the Prophet *Sallallahu Alayhi Wa Sallam* said,

عَنْ أَبِي مَالِكٍ الأَشْعَرِيِّ رضي الله عنه قالَ :قَالَ رسولُ الله صلى الله عليه وسلمَ:

الطَّهورُ شَطْرُ الإِيمَانِ. (رواه مسلم)

"Abu Malik Al-Asharee *Radhi Allahu anhu* said, "The Prophet *Sallallahu Alayhi Wa Sallam* said, "*Purity is half of Iman (faith)*" (Related by Muslim). Why did the Prophet *Sallallahu Alayhi Wa Sallam* say so? According to Al-Qazwini (2013), it is because iman will cleanse the inner filthy, meanwhile purity will cleanse the outer one. Wisdom or *hikmah* of the two hadiths is that the Prophet *Sallallahu Alayhi Wa Sallam* is the only one who knows the best matter for his *umma* with *shariah* and duties that are applied by him.

The hadiths above show that in the period of the Prophet *Sallallahu Alayhi Wa Sallam* it was used volume unit called *Qullah*. It was still used by Arabians about two centuries after the death of the Prophet *Sallallahu Alayhi Wa Sallam* until they substitute it with another unit so-called *rithel.* Meanwhile, recently many Arabians even do not know if they are asked a question about the amount of 1 *rithel* in a liter due to they also use the standard measure which holds nowadays (Thoriq, 2014). If Arabians who are now alive common to the unit so how about those who are not Arabians *('ajam)*. The main reason motivating us to write this article is not only to ethnics but also to the practice of shariah as the most important thing.

The question about unit *Qullah* which is related to mathematics is the conversion of the unit in the period of the Prophet *Sallallahu Alayhi Wa Sallam* into the international volume standard holding. It is the case to explore in this article. Thus it can be concluded that the hadith potentially contributes to mathematics educators to get insight mathematically about the measure of 2 *Qullahs* in volume unit holding in currently to ease people in practicing it, without any hesitancy.

RESEARCH METHOD
1. Research Type

This was library research. The aim of this was to present integrated Islamic and mathematics histories in terms of the unit *Qullah* in the measurement of volume in the period of the Prophet Muhammad *Sallallahu Alayhi Wa Sallam.*

2. Data Collection

The data were collected by exploring relevant references and literature. The main reference within this research was a book of Islamic jurisprudence, especially *fiqh taharah*, discussing the use of water with the unit *Qullah*. Besides, there was an article about the history of mathematics. Also, we referred to a historical book of Islamic civilization. The supporting references were the explanation of Islamic scholars related to the unit *Qullah* mathematically and other relevant references. Further, the literature was then analyzed and reviewed their relevance to the case discussed.

RESULTS AND DISCUSSION
1. The Unit *Qullah* and Islamic Jurisprudence *(Fiqh)* Perspectives on *Qullah*

Etymologically, *Qullah* is the name for something upside and high. It then became a name for a certain pitcher that ancient Arabians utilized to fulfill their needs, because of its high and big in measure. Some said that the word *Qullah* originated from the habits of men in bringing a pitcher by lifting it highly. Terminologically, *Qullah* is a volume with a certain measure in Islamic jurisprudence, namely 250 rithels (Bejo, 2015). Briefly, *Qullah* is a standard for a derived unit of the volume of water that ancient Arabians utilized (Thoriq, 2014).

Since the period of the Prophet *Sallallahu Alayhi Wa Sallam,* Arabians used the measure of *qullah* as a measuring-rod in determining a measure. The Prophet *Sallallahu Alayhi Wa Sallam* said in a hadith (meaning), "If the water is more than 2 *Qullahs,* it will not become filthy." (Related by the four imams). Within the explanation of the hadith, Bejo (2015) states that in Islamic jurisprudence perspective, if the amount of water is sufficient to fill 2 *Qullahs* (*Qullataeen*), it will not become filthy with the impurity getting inside as long as no changes in one of its characteristics that is smell, taste, and color. On the contrary, if the volume of water is insufficient to fill 2 *Qullahs* and there is an impurity like carrion of a rat falling inside, then it will become filthy and someone is legally unable to use it for purification. Nevertheless, if the water 2 *Qullahs* contains a lot of filthy resulting in the change in its smell, taste, or color, it means it is filthy totally. Fortunately, if the smell, taste, or color of the impurity is lost, the water 2 *Qullahs* is back to be pure.

Meanwhile, al-Utsaimin (2014) states that in essentials if the water is sufficient to fill 2 *Qullahs* and then there is an impurity getting inside, it becomes filthy. But, if nothing changes at all, it remains considered as pure. If the water is insufficient to fill 2 *Qullahs*, someone can judge it in the same way. If there is an impurity getting inside and then changing its condition, it becomes filthy. If nothing changes, it remains pure. But for sure, if the amount of water is little with a lot of impurities getting inside, it tends to change it. Or, if there is a lot of impurities getting inside the water less than 2 *Qullahs*, it most likely changes.

2. The Measure of the Volume 2 *Qullahs*

When measuring the volume of *Qullah*, the ancient Islamic scholars compared it to the less volume measure. Most of them measured the volume of *Qullah* by comparing it to rithel. There were some scholars' opinions about the volume of *Qullah*. Three types of rithels that scholars used in comparing *Qullah*, that is, rithels of Baghdad, Damascus, and Egypt. Within Baghdad's rithel, $1\ Qullah = 250\ rithels$. Whereas in Damascus' rithel, $1\ Qullah = 10\frac{1}{3}\ rithels$. However in Egypt's rithel, $1\ Qullah = 446\frac{3}{7}\ rithels$. Of the three *rithels*, the most common used by the scholars of Islamic jurisprudence is Baghdad's *rithel*. Thus, if $1\ Qullah = 250\ rithels$, then $2\ Qullah = 500\ rithels$. It can be concluded that the amount of water that can hold impurity back is about 500 rithels (Bejo, 2015).

Regarding the measure *Qullah* in nowadays, there were differences of opinion among Islamic scholars. Some of them say that the volume of 2 *Qullahs* is 160.5 liters. The others say 192, 204, 270, or 307 liters. Another says 2 *Qullahs* is 461.754 liter (Bejo, 2015).

Bejo (2015) quotes from the book of *Sharah al-Mahalee* that if 2 *Qullahs* is cubical, the measure of its edges is 1.25 cubits *(dzira')*. The volume of the cube is s^3, where s is the length of the edge. Thus, the formula for finding the volume of 2 *Qullahs* is:

$$V_{2\ Qullahs} = (1.25 \text{ cubits})^3$$

If the form of the pitcher is cylindrical like a well, the length of the diameter *(d)* of the object fulfilling 2 *Qullahs* is 1 cubit and the height *(h)* 2.5 cubits. The formula for the volume of the cylinder is:

$$V = \pi r^2 h$$

Based on the formula, the volume of 2 *Qullahs* for a cylinder is also as follows.

$$V = \pi r^2 h$$

In mathematics, the diameter *(d)* of a circle is equal to 2 times the length of its radius *(r)*. It can be written mathematically like this: $d = 2r$ or $r = \frac{1}{2}d$. Thus,

$$V_{2\ Qullahs\ of\ cylinder} = \pi \left(\frac{1}{2}d\right)^2 h = \pi \frac{1}{4}d^2 h = \frac{1}{4}\pi d^2 h$$

Because of the length of diameter $d = 1$ cubit, and the height $h = 2.5$ cubits, so:

$$V_{2\ Qullahs} = \frac{1}{4}\pi \times (1 \text{ cubit})^2 \times 2.5 \text{ cubits, where } \pi \approx \frac{22}{7} \text{ or } \pi \approx 3{,}14$$

3. A Question Example and the Solution: *Conversion of the Volume of 2 Qullahs into International Standard Unit*

The following is shown a question example and the solution by applying the aforementioned formula.

> **Example**
> Given the length of one cubit of a cube is 46.2 cm. How many liters of 2 *Qullahs* of water?

Solution:
Given:

    1 cubit = 46,2 cm

Asked:

    How many liters of 2 *Qullahs* of water?

Answered:

    The main formula is:

$$V_{2\ Qullahs} = (1.25 \text{ cubits})^3$$
$$1 \text{ cubit} = 46.2 \text{ cm}$$
$$1 \text{ dm}^3 \quad = 1 \text{ Liter}$$
$$1.25 \text{ cubits} = 1 \text{ cubit} + 0.25 \text{ cubits}$$

Since the *Qullah* is cubical with the edge is 1.25 cubits in length, whereas the well-known length of the edge in the international standard unit now is 1 cubit = 46.2 cm, so it is also subsequently necessary to convert the length of 0.25 cubits into the international standard unit. The remainder of the length of the edge 0.25 cubits is then converted into the international standard unit in the following way.

$$0.25 \text{ cubits} = \frac{1}{4} \times 46.2 \text{ cm} = 11.55 \text{ cm}$$

So, 1.25 cubits = 1 cubit + 0.25 cubits = 46.2 cm + 11.55 cm = 57.75 cm

Next, we substitute the value into the formula for the volume of 2 *Qullahs*:

$$V_{2\ Qullahs} = (1.25 \text{ cubits})^3$$
$$= (57.75 \text{ cm})^3$$
$$= 192{,}599.859375 \text{ cm}^3$$

Since 1 cm^3 = 0.001 dm^3, if it is changed into dm^3, it will be:

$$V_{2\ Qullahs} = 192{,}599.859375 \text{ cm}^3$$
$$= 192.599859375 \text{ dm}^3$$
$$= 192.6 \text{ dm}^3$$

Since 1 dm^3 = 1 liter, so

$$V_{2\ Qullahs} = 192.6 \text{ dm}^3$$
$$= 192.6 \text{ liters}$$

Thus, the volume of 2 *Qullahs* is 192.6 liters.

The result of the computation above is following the note that 2 *Qullahs* = 192 liters so that 1 *qullah* is equal to 96 liters. Since the determination of the measure of *Qullah* uses the computation of the measures of length and volume, so there is a difference 0.6 liter from the measure of the cube above and that of previous *Qullah*. However, it is not something problematical, because in the measure of the cube contains the measure of 2 *Qullahs*, even more than that about 0.6 liters.

CONCLUSIONS

Based on the outline above, it is acknowledged that there are differences of opinion among either the ancient or contemporary scholars regarding the measure of *Qullah*. Some of them say that the volume of 2 *Qullahs* is 160.5 liters. The others say 192, 204, 270, or 307 liters. Another says 2 *Qullahs* is 461.754 liter. Nevertheless, of the various opinions, it takes the opinion stating that 2 *Qullahs* is $\pm$ 192 liters, so that 1 *Qullah* is $\pm$ 96 liters, and it is in line with the result of

computation mathematically based on the explanation of the scholars of contemporary Islamic jurisprudence *(fiqh)*.

Mathematics educators are suggested to teach the topic of the volume of solid figures based on the mathematics history by showing the variation of using volume unit, either standard like meter, or nonstandard like *Qullah*. It is expected to be like Lucia Grugnetti, the researcher of mathematics history in learning, states that mathematics teaching in historical context can provide students with more chances to evaluate their problem-solving strategy (Deakin, 2001). Meanwhile, the most significant purpose of all is to make students recognize the treasure of science in Islamic civilization age, love Islamic teachings, and motivated to continuously study and practice the Qur'an as well as the Hadith. At last, it is expected to produce students with faith *(iman)*, creed *(taqwa),* and good deeds *(akhlaqul kareemah)* under the goal of national education through mathematics learning.

REFERENCES

Al-Qazwini, Abu Ja'far Umar. 2013. *Syarah 77 Cabang Iman: Imam Al-Baihaqi* (Translation). Bekasi: Darul Falah.

Al-Utsaimin, Syaikh Muhammad bin Shalih. 2014. *Fikih Thaharah: Pembahasan dari Kitab Fath Dzi Al-Jalal wa Al-Ikram Syarah Bulughul Maram* (Translation). Jakarta: Darus Sunnah Press.

Basya, Ahmad Fuad. 2015. *Sumbangan Keilmuan Islam pada Dunia* (Translation). Jakarta: Pustaka Al-Kautsar.

Bejo, Mohammad. 2015. *Asyiknya Berhitung dalam Fikih: Mengubah Kesan Rumit dalam Fikih Menjadi Sederhana dan Mengasyikkan.* Yogyakarta: Pro-U Media.

Deakin, M. A. B. 2001. Katz, Victor (Ed.) Using History to Teach Mathematics: An International Perspective. *ZDM*, 33 (5), 137-138.

Thoriq, Ahmad Syahrin. 2014. *Air 2 Qullah*. Retrieved from: http://www.konsultasi slam.com/2011/11/air-2-qullah.html. (Wednesday, July 3, 2019)