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Numerical Solution of Diabetes Mellitus Model without Genetic Factors with Treatment using Runge Kutta Method

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Abstract. This study discusses the numerical solution of the model analysis of diabetes mellitus without genetic factors with treatment using the fourth order Runge-Kutta. The mathematical model in the form of a differential equations system that includes S *(Susceptible), E (Exposed), I (Infected),* and I_T *(Infected with Treatment)* which has been simplified into the total population class (N), the latent population (E), the patient population without treatment (I) and the patient population with treatment (I_T) as the initial value and the value of A, μ, δ_1, δ_2, α, β as parameters were resolved numerically using the fourth order Runge-Kutta method and performed as many iteration with interval time or $h = 0,01$ years. The data taken in Makassar City for each class population. Initial and parameters values are subtituted into numerical solutions to the model which are then simulated using Maple, indicating that at t = 5 years the magnitude of the entire population class   (N) = 195.216, the latent population (E) = 31, the patient population without treatment (I) = 4.836 and the patient population with treatment (I_T) = 1.454. The results concluded that the magnitude of the value for the rate of each class of population in the next five years decreased due to the population dead and/or move the next class of the population.

Keywords: Diabetes mellitus, fourth order Runge-Kutta, numerical solution

1. Introduction
Mathematical models are mathematical representations that result from mathematical modeling. Mathematical modeling is a process of representing and explaining real-world problems in mathematical statements [1]. Many problems in the world related to mathematics can be formed into differential equations systems. One of them is a disease that is increasingly suffered by the world's population and is increasing every year, namely diabetes mellitus (DM).

According to the results of basic health research in 2013, the prevalence of diabetes in South Sulawesi which was diagnosed by doctors was 1.6%. Diabetes mellitus that is diagnosed by a doctor or based on symptoms is 3.4%. The prevalence of diabetes diagnosed by a doctor or based on the symptoms of Makassar City occupies the second highest position [2]. Diabetes mellitus is a hereditary disease, although that does not mean that this disease will definitely decline in children. Although both parents suffer from diabetes mellitus, sometimes their children do not have diabetes mellitus [3]. The results of the research of Henrita, E (sales manager of health zone talents) as much as 80% of people

with diabetes mellitus are not due to hereditary factors but rather due to factors of food and beverage consumption patterns [4].

    Research on mathematical models on the spread of infectious diseases has been carried out by [5][6][7][8][9][10], this study provides a numerical solution of the diabetes mellitus model using the fourth order Runge-Kutta method as in the method [11] namely the model. Mathematics of diabetes mellitus without genetic factors with treatment. This fourth-order Runge-Kutta method provides higher accuracy results in calculations and rounding [12].

    The system of differential equations in the mathematical model of diabetes mellitus without genetic factors with treatment will be solved numerically using the Runge-Kutta method and then simulated the model that has been numerically completed with secondary data according to the population class in the model obtained in Makassar City in the form of initial value and parameter value given.

2. Runge-Kutta Method

The general form of the Runge-Kutta method is as in the following equation (1):

$$x_{i+1} = x_i + \Phi(t_i x_i h)h \tag{1}$$

with $\Phi(t_i x_i h)$ is an increase function which is the mean slope at the interval and is used to extrapolate from the old value x_i to the new value x_{i+1} throughout the interval h. The addition function can be written in general form as in the following equation (2):

$$\Phi = a_1 k_1 + a_2 k_2 + \cdots + a_n k_n \tag{2}$$

with a is a constant and k is:

$$k_1 = f(t_i x_i) \tag{3}$$
$$k_2 = f(t_i + p_i h, x_i + q_{11} k_1, h) \tag{4}$$
$$k_3 = f(t_i + p_i h, x_i + q_{21} k_1 h + q_{22} k_2 h) \tag{5}$$
$$.$$
$$.$$
$$.$$
$$k_n = f(t_i + p_{n-1} h, x_i + q_{n-1,2} k_1 h + q_{n-1,2} k_2 h + \cdots + q_{n-1,n-1} k_{n-1} h)$$

with p and q are constants. The value of k indicates the sequential relationship. The value of k_1 appears in equation (4), both of which also appear in equation (5), and so on [5].

3. Runge-Kutta Fourth Order Method

The fourth order Runge-Kutta method has the form as in the following equation (6) [5]:

$$x_{i+1} = x_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \tag{6}$$

with,

$$k_1 = f(t_i x_i)$$
$$k_2 = f\left(t_i + \frac{1}{2}h, x_i + \frac{1}{2}k_1 h\right)$$
$$k_3 = f\left(t_i + \frac{1}{2}h, x_i + \frac{1}{2}k_2 h\right)$$
$$k_4 = f(t_i + h, x_i + k_3 h)$$

4. Model of Diabetes Mellitus without Genetic Factors with Treatment

In this case, the treatment of patients with diabetes mellitus was carried out. So that the $SEII_T$ mathematical model was obtained in diabetes mellitus without genetic factors with treatment. Normal individuals (not yet affected by diabetes) are included in the Susceptible class, individuals who have bad habits, decreased insulin hormone and increased blood glucose are included in the Exposed class (latent population), and infected population are classified into two classes, namely infected class who

gets treatment (I_T) and infected class without treatment (I) [6]. Models can be seen in the transfer diagram as shown in the following:
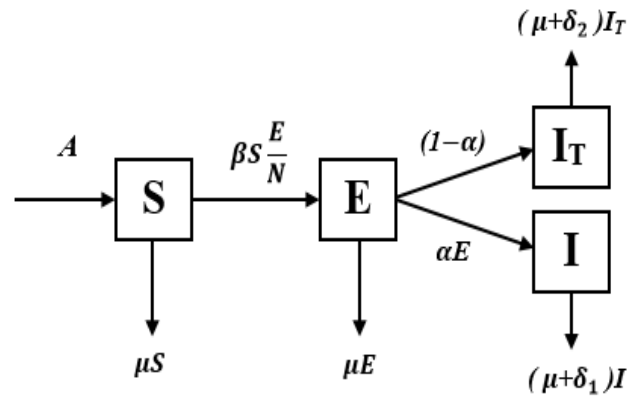

Figure 1. Diagram of mathematical model transfer of diabetes without genetic factors with treatment

Based on Figure 1, a mathematical model in the form of a system of ordinary differential equations which contains variables S, E, I, I_T, and N. N (t) states the number of population at t. So the system of differential equations of Figure 1 is as in equation (7) to (11) below:

$$\frac{dS}{dt} = A - \mu S - \beta S \frac{E}{N}, \tag{7}$$

$$\frac{dE}{dt} = \beta S \frac{E}{N}, -\mu E - E, \tag{8}$$

$$\frac{dI}{dt} = \alpha E - (\mu + \delta_1)I, \tag{9}$$

$$\frac{dI_T}{dt} = (1 - \alpha)E - (\mu + \delta_2)I_T, \tag{10}$$

$$N = S + E + I + I_T. \tag{11}$$

Equation (9) to (11) is obtained $\frac{dN}{dt} = A - \mu N - \delta_1 I - \delta_2 I_T$.

Because N = S + E + I + I_T, then $\frac{dS}{dt} = \frac{dE}{dt} - \frac{dI}{dt} - \frac{dI_T}{dt}$, so a simpler system is obtained in equation (12) to (15) as follows:

$$\frac{dN}{dt} = A - \mu N - \delta_1 I - \delta_2 I_T, \tag{12}$$

$$\frac{dE}{dt} = \beta(N - E - I - I_T)\frac{E}{N} - \mu E - E \tag{13}$$

$$\frac{dI}{dt} = \alpha E - (\mu + \delta_1)I, \tag{14}$$

$$\frac{dI_T}{dt} = (1 - \alpha)E - (\mu + \delta_2)I_T. \tag{15}$$

$$\frac{dI_T}{dt} = (1 - \alpha)E - (\mu + \delta_2)I_T. \tag{16}$$

5. Research Method

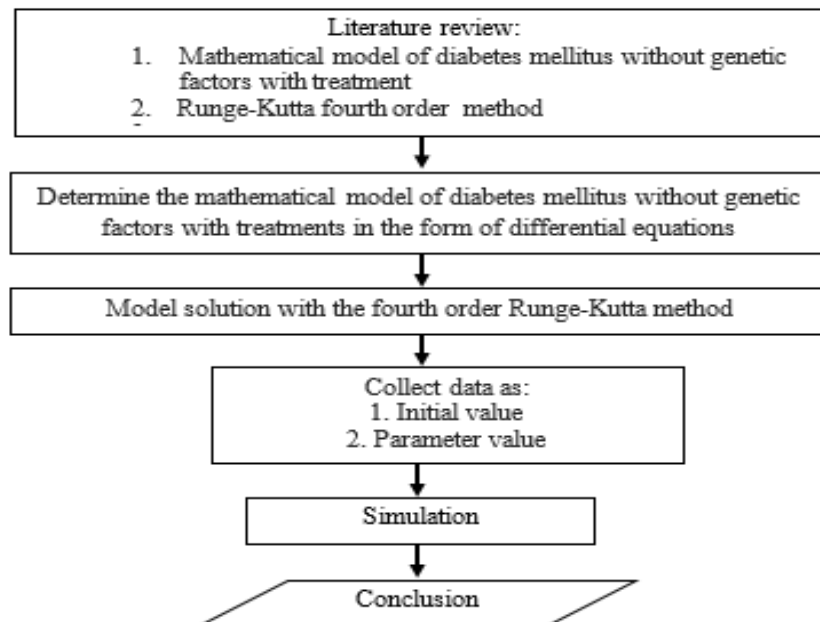This Mathematical modeling of SII for DM is a theoretical study.

Research Scheme

Figure 2. Problem-solving scheme

6. Results and Discussion

6.1. The model solution with the fourth order Runge-Kutta method

Equations (12) to Eq (15) will be solved using the fourth order Runge-Kutta method by equation (6). The model of diabetes mellitus without genetic factors with the treatment to be completed is a system of equations, so there is no need to change it. Equations (12) to Eq (15) are substituted in the fourth order Runge-Kutta equation so that the following equations (17) to Eq (20) are obtained:

$$N_{i+1} = N_i + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4) \tag{17}$$

$$E_{i+1} = E_i + \frac{1}{6}h(l_1 + 2l_2 + 2l_3 + l_4) \tag{18}$$

$$I_{i+1} = I_i + \frac{1}{6}h(m_1 + 2m_2 + 2m_3 + m_4) \tag{19}$$

$$I_{T_{i+1}} = I_{T_i} + \frac{1}{6}h(n_1 + 2n_2 + 2n_3 + n_4) \tag{20}$$

with,

$$k_1 = A - \mu N_i - \delta_1 I_i - \delta_2 I_{T_i}$$

$$l_1 = \beta\left(N_i - E_i - I_i - I_{T_i}\right)\frac{E_i}{N_i} - \mu E_i - E_i$$

$$m_1 = \alpha E_i - (\mu + \delta_1)I_i$$

$$n_1 = (1 - \alpha)E_i - (\mu + \delta_2)I_{T_i}$$

$$k_2 = A - \mu(N_i + k_1\frac{h}{2}) - \delta_1(I_i + m_1\frac{h}{2}) - \delta_2(I_{T_i} + n_1\frac{h}{2})$$

$$l_2 = \beta\left((N_i + k_1\frac{h}{2}) - (E_i + l_1\frac{h}{2}) - (I_i + m_1\frac{h}{2}) - (I_{T_i} + n_1\frac{h}{2})\right)\left(\frac{E_i + l_1\frac{h}{2}}{N_i + k_1\frac{h}{2}}\right) - \mu(E_i + l_1\frac{h}{2}) - (E_i + l_1\frac{h}{2})$$

$$m_2 = \alpha(E_i + l_1\frac{h}{2}) - (\mu + \delta_1)(I_i + m_1\frac{h}{2})$$

$$n_2 = (1 - \alpha)(E_i + l_1 \tfrac{h}{2}) - (\mu + \delta_2)(I_{T_i} + n_1 \tfrac{h}{2})$$

$$k_3 = A - \mu(N_i + k_2 \tfrac{h}{2}) - \delta_1(I_i + m_2 \tfrac{h}{2}) - \delta_2(I_{T_i} + n_2 \tfrac{h}{2})$$

$$l_3 = \beta \left((N_i + k_2 \tfrac{h}{2}) - (E_i + l_2 \tfrac{h}{2}) - (I_i + m_2 \tfrac{h}{2}) - (I_{T_i} + n_2 \tfrac{h}{2}) \right) \left(\frac{E_i + l_2 \tfrac{h}{2}}{N_i + k_2 \tfrac{h}{2}} \right) - \mu(E_i + l_2 \tfrac{h}{2}) - (E_i + l_2 \tfrac{h}{2})$$

$$m_3 = \alpha(E_i + l_2 \tfrac{h}{2}) - (\mu + \delta_1)(I_i + m_2 \tfrac{h}{2})$$

$$n_3 = (1 - \alpha)(E_i + l_2 \tfrac{h}{2}) - (\mu + \delta_2)(I_{T_i} + n_2 \tfrac{h}{2})$$

$$k_4 = A - \mu(N_i + k_3 h) - \delta_1(I_i + m_3 h) - \delta_2(I_{T_i} + n_3 h)$$

$$l_4 = \beta \left((N_i + k_3 h) - (E_i + l_3 h) - (I_i + m_3 h) - (I_{T_i} + n_3 h) \right) \left(\frac{E_i + l_1 h}{N_i + k_1 h} \right) - \mu(E_i + l_3 h) - (E_i + l_3 h)$$

$$m_4 = \alpha(E_i + l_3 h) - (\mu + \delta_1)(I_i + m_3 h)$$

$$n_4 = (1 - \alpha)(E_i + l_3 h) - (\mu + \delta_2)(I_{T_i} + n_3 h)$$

6.2. Numerical simulation of the model using the fourth order Runge-Kutta Method

From the data obtained at the Makassar City Health Office and Central of Statistics, the initial variable values and parameter values that will be used in the simulation of the numerical solution of the diabetes mellitus model without genetic factors with treatment using the fourth order Runge-Kutta method have been obtained. The initial variable values data can be seen in Table 1.

Table 1. Initial value

Variable	Values
$N_{(0)}$	395.406
$E_{(0)}$	9.304
$I_{(0)}$	4.779
$I_{T_{(0)}}$	3.426

The recruitment rate in the selected population A = 2 natural mortality rate is 72.1 years, then $\mu = \frac{1}{\text{life expectancy}} = \frac{1}{72,1} = 0,13869$, $\beta = 0,0009$, it means that on average there are 9 individuals susceptible that becomes latent when there are 1000 susceptible individuals who come into contact with latent individuals, the latent rate of movement of the individual becomes infected $\alpha = \frac{\text{number of diabetes}}{\text{latent number}} = \frac{8.205}{9.304} = 0,88187$, the rate of death due to diabetes without treatment $\delta_1 = \frac{\text{number of diabetes deaths}}{\text{Number of diabetes without treatment}} = \frac{318}{4.779} = 0,06654$ and the rate of death due to diabetes with the influence of treatment $\delta_2 = \frac{\text{number of diabetes deaths}}{\text{Number of diabetes with treatment}} = \frac{318}{3.426} = 0,09281$. The following parameter values in Table 2.

Table 2. Parameter value

Parameter	Value
A	2
μ	0,13869
δ_1	0,06654
δ_2	0,09281
α	0,88187
β	0,0009

The simulation is done by substituting the initial values and values of the parameters given in Tables 1 and 2 into equations (17) to Eq (20) which are numerical solutions to the model of diabetes mellitus without genetic factors with treatment using the Runge-Kutta method. The next four will be illustrated through graph plots using the Maple.

The time interval or step distance is used $h = 0,01$. Then given $N_i = N_{(0)}$, $E_i = E_{(0)}$, $I_i = I_{(0)}$, $I_{T i} = I_{T (0)}$ as the initial value so that the solution results in numerical models of diabetes mellitus without genetic factors with treatment using the fourth order Runge-Kutta four-order method as follows:

$$k_1 = -55.472,81986$$
$$l_1 = -10.586,36895$$
$$m_1 = 7.224,12431$$
$$n_1 = 305,9625$$
$$k_2 = -55.436,89768$$
$$l_2 = -10.526,14146$$
$$m_2 = 7.170,032269$$
$$n_2 = 299,3555293$$
$$k_3 = -55.436,90153$$
$$l_3 = -10.526,48410$$
$$m_3 = 7.170,353340$$
$$n_3 = 299,3987509$$
$$k_4 = -55.400,98344$$
$$l_4 = -10.466,59535$$
$$m_4 = 7.116,578689$$
$$n_4 = 292,8344758$$

with a substitute ,the value of k_1 to k_4, l_1 to l_4, m_1 to m_4 and n_1 to n_4 in equation (17) to (20) is obtained numerical solution model of diabetes mellitus without genetic factors with treatment using the method of Runge-Kutta fourth order as follows:

$N_{(0+1)} = N_0 + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4)$

$N_{(1)} = 395.406 + \left(\frac{1}{6}\right)(0,01)(-55.472,81986 + 2(-55.436,89768) + 2(-55.436,90153) +$
$\qquad (-55.400,98344)) = \mathbf{394.851,631}$

$E_{(0+1)} = E_0 + \frac{1}{6}h(l_1 + 2l_2 + 2l_3 + l_4)$

$E_{(1)} = 9.304 + \left(\frac{1}{6}\right)(0,01)(-10.586,36895 + 2(-10.526,14146) + 2(-10.526,48410) +$
$\qquad (-10.466,59535)) = \mathbf{9.198,737306}$

$I_{(0+1)} = I_0 + \frac{1}{6}h(m_1 + 2m_2 + 2m_3 + m_4)$

$I_{(1)} = 4.779 + \left(\frac{1}{6}\right)(0,01)(7.224,12431 + 2(7.170,032269) + 2(7.170,353340) + 7.116,578689)$
$\qquad = \mathbf{4.850,702457}$

$I_{T (0+1)} = I_{T 0} + \frac{1}{6}h(n_1 + 2n_2 + 2n_3 + n_4)$

$I_{T (1)} = 3.426 + \left(\frac{1}{6}\right)(0,01)(305,96252 + 2(299,3555293) + 2(299,3987509) + 292,8344758)$
$\qquad = \mathbf{3.428,993843}$

So, at t = 0.01 the magnitude of the number of each population class (N) is 398,852, the latent population rate (E) is 9,199, the rate of population of diabetes without genetic factors without treatment (I) is 4,851 and the rate diabetics without genetic factors with treatment (I_T) are 3,429. Then the same thing is done for the next iteration with N_i, E_i, I_i, $I_{T i}$ where $i = 1, 2, 3, ...,$ etc. as the initial

value. Iteration results up to t = 5 years for the rate of each population class will be shown in the graph plot as in Figure 3 through Figure 6 below.
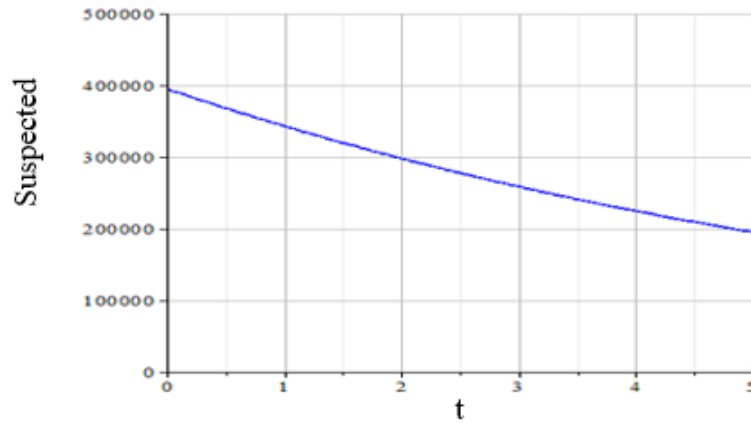

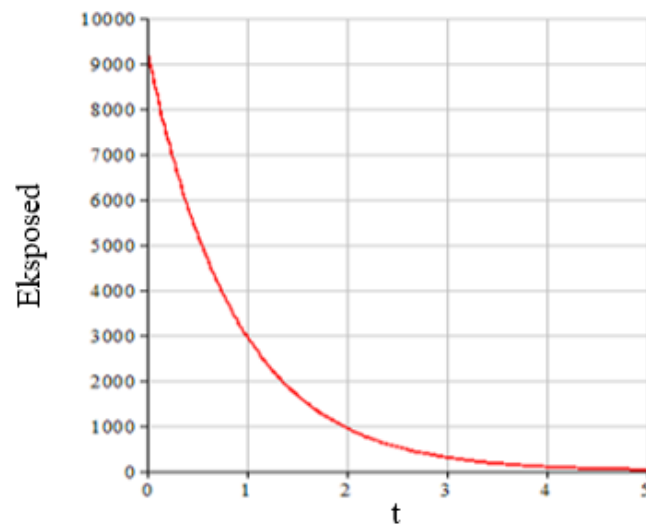**Figure 3.** The iteration graph of the total population class (N)

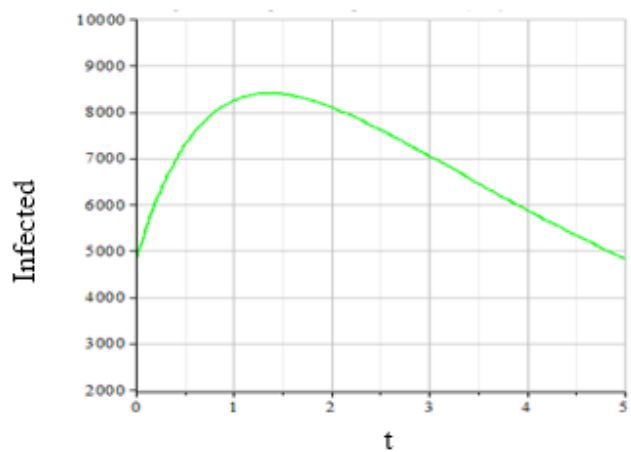**Figure 4.** Graph of latent population iteration (E)

Figure 5. Graph of the population without diabetes treatment iteration (I)
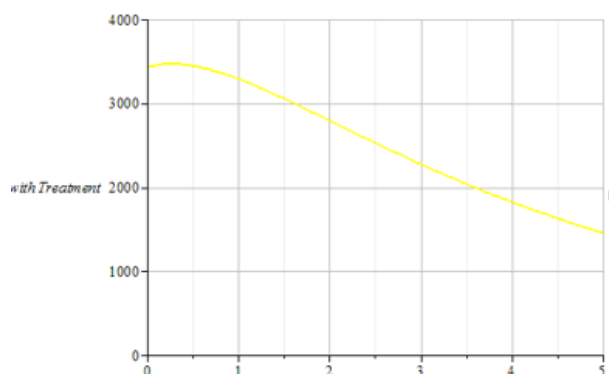
Figure 6. Graph of the population with diabetes mellitus treatment iteration (I_T)

Based on the results of the iteration shown both in the plot graph in Figures 3 to 6 it can be seen that the magnitude of the rate of the total population class (N) has decreased due to the move to the next population class and or the existence of a dead population, the magnitude of the total number of population classes (N) in the 500th iteration or at t = 5 is 195,216 people. For the latent population rate (E) which has decreased due to the number of latent population classes that move into diabetic populations without genetic factors either with care or without treatment and or the existence of a dead population, the latent population value (E) in the 500th iteration or when t = 5 is 31 people.

According to the rate of people class with diabetes mellitus without genetic factors either without the influence of treatment (I) or with the influence of treatment (I_T) had experienced an increase and then continued to decline until the next time. Classes with diabetes mellitus without treatment influence experience the highest increase in the 140th iteration or at t = 1.4 with a value of 8,398 people, then continue to decline until the 500th iteration or at t = 5 with a value of 4,836 people. Similar to population class I, the I_T population class had an increase until the 30th iteration or at t = 0.3 with a value of 3,465 people and then continued to decline until the 500th iteration or at t = 5 with the value 1,454 people. The decline in the rate of the class of people with diabetes mellitus without genetic factors both with care and without treatment due to a population that dies.

Based on the above description, it can be concluded that the magnitude of the value of the entire population class when t = 5 has decreased due to the move to the next population class and or the existence of a dead population.

7. Conclusion

The solution of diabetes mellitus without genetic factors with numerical treatment model using the fourth order Runge-Kutta method with interval time or h = 0.01. The results for the 1st iteration are as follows: $N_{(1)} = \mathbf{398.851,631}$; $E_{(1)} = \mathbf{9.198,737306}$; $I_{(1)} = \mathbf{4.850,702457}$ and $I_{T_{(1)}} = \mathbf{3.428,993843}$.

By completing a simulation model of diabetes mellitus without genetic factors with numerically resolved treatments using the fourth-order Runge-Kutta method in Makassar City we can predict the rate of population in the model of diabetes mellitus without genetic factors with treatment for the next five years based on data in 2016. Among them, the magnitude of the total number of classes of pollution (N) at t = 5 is 195,216, the amount of the latent population (E) at t = 5 is 31, the magnitude of the population without diabetes treatment (I) at t = 5 is 4,836, and the rate of population of diabetics with treatment (I) at t = 5 diabetics with treatment (I_T) is 1,454.

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