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The Application of Algebraic Methods in Balanced Incomplete Block Design

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Abstract. A balanced incomplete block design contains finite set P with v elements called varieties and a collection of set B consisting of b sets called blocks to qualify: (i). each block contains k varieties, (ii). Each variety occurs in every r blocks, (iii). Each pair of varieties occurs in exactly λ block. In this paper we discuss the use of algebraic method to construct such designs.

Keywords— *Algebraic, BIBD*

1. Introduction

In certain experiments using randomized block design (RBD), we may not be able to run all treatment combinations in each block. Situations like this usually occur because of shortages experimental equipment or facilities or the physical size of the block. To solve this problem, we can use a randomized block design where each treatment does not need to exist in every block. The design is known as balanced incomplete block design (BIBD). If all treatment comparisons are equally important, the treatment combinations used in each block should be selected in a balanced manner, so that any pair of treatments occur together the same number of times as any other pair. Thus, a BIBD is an incomplete block design in which any two treatments appear together an equal number of times. Suppose that there are a treatments and each block can hold exactly k ($k < a$) treatments, then a BIBD

may be constructed by taking $\binom{a}{k}$ block and assigning a different combinations of treatments to each

block. In this paper, we will apply the methods of modern algebra to construct a BIBD. Detailed discussion of statistical analysis of BIBD will not be discussed in this paper. Likewise, the algebraic method. We will only discuss the use of the methods of modern algebra at BIBD. For more details on the methods of modern algebra, see [1] and [2] and for a discussion of BIBD and its statistical analysis, see [3] and [4]

2. Algebraic Method of BIBD

Definition 1

A BIBD with parameters (v, b, r, k, λ) is a pair (P, B) with the following properties:

- (i). P is a set with v elements.
- (ii). $B = (B_1, \dots, B_b)$ is a subset of the power set $\wp(P)$ with b elements.
- (iii). each B_i has exactly k elements, where $k < v$.



(iv). each unordered pair (p, q) with $p, q \in P, p \neq q$, occurs in exactly λ elements of B .

The sets B_1, \dots, B_b are called the block of BIBD. Each $a \in P$ occurs in exactly r sets of B . Such a BIBD is also called a (v, b, r, k, λ) configuration or 2 - 2 - (v, k, λ) tactical configuration or design. The term "balanced" indicates that each pair of elements appears in exactly the same number of blocks, while the term "incomplete" means that each block contains less than v elements. A BIBD is symmetric if $v = b$.

Definition 2

The incidence of matrix a (v, b, r, k, λ) configuration is the $v \times b$ matrix $A = (a_{ij})$, where

$$a_{ij} = \begin{cases} 1 & \text{if } i \in B_j \\ 0 & \text{otherwise} \end{cases}$$

Here i denotes the i th element of the configuration.

Some properties of incidence matrices:

Theorem 1

Let A is the incidence matrix of a (v, b, r, k, λ) configuration, then:

$$(i). AA^T = (r - \lambda)I_v + \lambda J_{vv}$$

$$(ii). \det(AA^T) = [r + (v - 1)\lambda](r - \lambda)^{v-1}$$

$$(iii). AJ_{bb} = rJ_{rb}$$

$$(iv). J_{vv}A = kJ_{vb}$$

where J_{mn} is the $m \times n$ matrix with all entries equal to 1 and I_n is the $n \times n$ identity matrix.

Theorem 2

The following conditions are necessary for the existence of a BIBD with parameters v, b, r, k, λ :

$$(i). bk = rv$$

$$(ii). r(k - 1) = \lambda(v - 1)$$

$$(iii). b \geq v$$

Proof:

(i). we count of the number of ordered pairs consisting of a block and an element of the block as bk . Each of P must appear in r blocks B . Therefore, there are vr ordered pairs of an element and a block containing it. The numbers bk and vr count the number of elements of the same set, so $bk = vr$.

(ii). Follows by a similar argument

(iii). If $v > \lambda$ then A^T has rank v , (by Theorem 1 (ii)). On the other hand, since the rank of A can be at most b , and since $\text{rank } A \geq \text{rank } AA^T = v$, then $b \geq v$. ■

Example 1

Let $\{1, 2, 3, 4, 5, 6, 7\}$ be the set of elements (varieties) and $\{\{1, 2, 3\}, \{1, 5, 6\}, \{3, 4, 5\}, \{1, 4, 7\}, \{3, 6, 7\}, \{2, 5, 7\}, \{2, 4, 6\}\}$ is the set of the blocks. It is obtained a $(7, 7, 3, 3, 1)$ configuration. This is the so called Fano geometry, the simplest example of a finite projective plane over F_2 . The blocks are the set of points on lines and the element are 7 points of the plane.

There is a special type of algebraic structure, near rings, which can be used to construct block design.

Definition 3

A set N with operations $+$ and \cdot is near-ring provided that:

(i). $(N, +)$ is a group (not necessarily commutative)

(ii). (N, \cdot) is a semigroup.

(iii). $\forall n, n', n'' \in N: (n + n') \cdot n'' = n \cdot n'' + n' \cdot n''$.

Each ring is a near-ring. Two ring axioms does not exist in near-ring, which is commutative of addition and distributive properties.

Definition 4

Let N be a near-ring and $n_1, n_2 \in N$;

$$n_1 \equiv n_2 : \Leftrightarrow \forall n \in N, n \cdot n_1 = n \cdot n_2$$

Definition 5

The near-ring N is called planar if $|N| \geq 3$ and if all the equations $x \bullet a = x \bullet b + c$, ($a, b, c \in N$, a not mod of b) have exactly one solution $x \in N$.

Near-ring planar produce BIBD's with parameters v, b, r, k, λ and efficiency $E = \lambda v / rk$. This E is a number between 0 and 1 and it estimate, for the statistical analysis of experiments, the "quality" of the design: .BIBD's with efficiency $E \geq 0.75$ are usually considered to be "good" ones.

One get BIBD's from planar near rings in the following way: We define for $a \in N$, N a planar near-ring, $ga : N \rightarrow N, n \mapsto n \bullet a$ and from $G = \{ga \mid a \in N\}$. Call $a \in N$ as a "group-forming" if $a \bullet N$ is a subgroup of $(N, +)$. Let us also call all sets $a \bullet N + b$, ($a \in N^*, b \in N$) as a block. Then these blocks together with N as a set of points from a tactical configuration with parameters:

$$(v, b, r, k, \lambda) = \left(v, \frac{\alpha_1 v}{|G|} + \alpha_2 v, \alpha_1 + \alpha_2 |G|, |G|, \lambda \right) \text{ where } v = |N| \text{ and } \alpha_1(\alpha_2) \text{ denote the number of}$$

orbit of F under the group- $G \setminus \{0\}$ which consists entirely of group forming (nongroup forming, respectively). This tactical configuration is a BIBD if and only if all either all elements are group-forming or just 0 is group forming.

Example 2

We consider the problem of testing combinations of six out of ten fertilizers, each on three fields.. By Definition 1, we have to look for a BIBD with parameters $(v, 10, 6, 3, \lambda)$. By Theorem 2 (i) and (ii) we obtain $v = 5$ and $\lambda = 3$. Hence we are searching for a planar near- ring of order 5 and the near-ring N .

We construct the blocks $a \bullet N + b$, ($a \neq 0$):

- $1 \bullet N + 0 = 4 \bullet N + 0 = \{0, 1, 4\} := B_1$
- $1 \bullet N + 1 = 4 \bullet N + 1 = \{1, 2, 0\} := B_2$
- $1 \bullet N + 2 = 4 \bullet N + 2 = \{2, 3, 1\} := B_3$
- $1 \bullet N + 3 = 4 \bullet N + 3 = \{3, 4, 2\} := B_4$
- $1 \bullet N + 4 = 4 \bullet N + 4 = \{4, 0, 3\} := B_5$
- $2 \bullet N + 0 = 3 \bullet N + 0 = \{0, 2, 3\} := B_6$
- $2 \bullet N + 1 = 3 \bullet N + 1 = \{1, 3, 4\} := B_7$
- $2 \bullet N + 2 = 3 \bullet N + 2 = \{2, 4, 0\} := B_8$
- $2 \bullet N + 3 = 3 \bullet N + 3 = \{3, 0, 1\} := B_9$
- $2 \bullet N + 4 = 3 \bullet N + 4 = \{4, 1, 2\} := B_{10}$

It is clear that only 0-is group-forming. Parameters can also be obtained directly from:

- (i) $v = 5$ points (*ie*: 0, 1, 2, 3, 4)
- (ii) $b = 10$ blocks
- (iii) Each point lies in exactly $r = 6$ blocks
- (iv) Each block contains precisely $k = 3$ elements
- (v) Each pair of different points appears in $\lambda = 3$ blocks

To solve our fertilizer problem, we divide whole field into five parts which we call 0, 1, 2, 3, 4. Then we use fertilizers F_i on every field of the block B_i ($i = 1, 2, \dots, 10$).

Field 0	Field 1	Field 2	Field 3	Field 4
F_1	F_1			F_1
F_2	F_2	F_2		
	F_3	F_3	F_3	

		F_4	F_4	F_4
F_5			F_5	F_5
F_6		F_6	F_6	
	F_7		F_7	F_7
F_8		F_8		F_8
F_9	F_9		F_9	
	F_{10}	F_{10}		F_{10}

Then we obtain: Every field contains exactly 6 fertilizers, and every fertilizer is applied on three fields. Every pair of different fields have three fertilizers in common. The efficiency E of this BIBD is $E = (\lambda v/rk) = 0.83$

3. Conclusion

The method used to construct the BIBD is still possible if the number of elements of P is not too much. The problems will arise if in constructing the BIBD there are elements of P that is too large. It will be very difficult to do. For situation which P is too large, we can use graph theory as in [Budayasa, 2004].

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