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ON THE NON-NEGATIVITY OF PROBABILITY DENSITY FUNCTIONS

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Abstract

The definition of probability density function requires to be non-negative. Mathematical software is used to easily compute some statistical measures, such as, mean, variance, covariance, and correlation of several density functions of which the non-negativity properly is ignored. This paper will discuss some contradictions when such property of non-negativity is neglected.

Key Words: *Non-negative function, probability density function, variance, correlation.*

1. INTRODUCTION

The definition of probability density function (pdf) requires a function 10 be non-negative. There are two random variables common in constructing a probability density function, namely, discrete and continuous random variables. Mathematical software, Maple, is used to easily compute some statistical measures, such as, mean, variance, covariance, and correlation coefficient of several density functions of which the non-negativity property is ignored. This paper will discuss some contradictions when such property of nan-negativity is neglected. Furthermore, this paper will focus only on continuous random variable, in which an integral approach is used to calculate that some statistics measurements.

2. THEORY

2.1. PROBABILITY DENSITY FUNCTIONS

Several definitions of the concept of probability are as follows (Walpole, 2013):

a. A random variable is a variable that associates 2 real number with each element in the sample space;
b. If a sample space contains a finite number of possibilities or an unending sequence with us many elements as there are whole numbers, it is called a discrete sample space.
c. If a sample space contains an infinite number of possibilities equal to the number of points on a line segment, it is called a continuous sample space.

Continuous random variable X has a special distribution function called probability density function (pdf) and denoted as f(x). It has some special characteristics, such as:

a. $f(x) \geq 0$ for all $x \in \mathbf{R}$

b. $\int_{-\infty}^{+\infty} f(x)dx = 1$

c. $P(a<X<b) = \int_{a}^{b} f(x)dx$, for $a, b \in \mathbf{R}$.

Explanation: (a) nor-negative function property, (5) the probability value for sample space is 1, and (c) the probability value for a bounded interval. For computing some expectation values, namely, mean, variance, covariance, and correlation coefficient for 1 continuous random variable, the fallowing formulas are applied:

$\mu = E(x) = \int_{-\infty}^{\infty} xf(x)dx$ — mean of X

$Var(x) = \sigma_x^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$ — variance of X

$Cov(x,y) = \gamma_{xy} = E(xy) - E(x)E(y)$ — covariance of X & Y

$Corr(x,y) = \rho_{xy} = \frac{Cov(x,y)}{\sqrt{Var(x).Var(y)}}$ — correlation of X & Y

The results of computation will show that the variance values are always positive and correlation coefficient is $|\rho| \leq 1$.

In computation, we used mathematical software (Maple). This computation is performed to find statistic values, such as, mean, variance, covariance, and correlation coefficient. All these statistic values are used in comparing between the non-negative function and negative function.

The function f(x,y) is a joint probability density function of the continuous random variables X and Y if

a. $f(x,y) \geq 0$ for all $x, y \in \mathbf{R}$

b. $\iint_{-\infty}^{\infty} f(x,y)dxdy = 1$

c. $P[(X,Y) \in A] = \iint_A f(x,y)dxdy$, for any region A in the xy plane.

2.2. SOME STATISTIC MEASUREMENT OF PDF

This section shows two examples which satisfy the non-negative properties.

Example 1. In [4]: Joint probability density function of the random variables X and Y is

$$f(x,y) = \begin{cases} 6x; & 0 < x < 1, 0 < y < 1-x \\ 0; & \text{else where} \end{cases} \quad (1)$$

We can display the geometric form of the domain of the function as follows:

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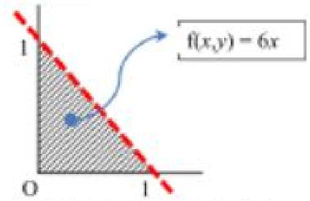

Figure 1.The domain of f(x,y) = 6x

The following is the maple syntaxes and results:

>f:=6*x; $f := 6\,x$ --- *this is the probability density function*

>mx:=int(int(x*f,y=0..1-x),x=0..1); $mx := \frac{1}{2}$ --- *this is mean of X OR E(x)*

>my:=int(int(y*f,y=0..1-x),x=0..1); $my := \frac{1}{4}$ --- *this is mean of Y OR E(y)*

>mxy:=int(int(x*y*f,y=0..1-x),x=0..1); $mxy := \frac{1}{10}$ --- *this is E(xy)*

>mx2:=int(int(x*x*f,y=0..1-x),x=0..1); $mx2 := \frac{3}{10}$ --- *this is* $E(x^2)$

>my2:=int(int(y*y*f,y=0..1-x),x=0..1); $my2 := \frac{1}{10}$ --- *this is* $E(y^2)$

>vx:=mx2-mx*mx; $vx := \frac{1}{20}$ --- *this is the variance of X OR var(x)*

>vy:=my2-my*my; $vy := \frac{3}{80}$ --- *this is the variance of Y OR var(y)*

>cxy:=mxy-mx*my; $cxy := \frac{-1}{40}$ --- *this is the covariance of X & Y OR cov(x,y)*

>rho:=cxy/sqrt(vx*vy); $\rho := -\frac{1}{3}\sqrt{3}$ --- *this is the correlation of X & Y*

>evalf(rho); -.5773502693 --- *this is the correlation of X & Y*

This results show that the variances of both random variables are positive and correlation coefficient $\rho = -0.577$ which lies in interval $-1 \le \rho \le 1$.

Example 2. The joint probability density function of the random variables X and Y is

$$f(x,y) = \begin{cases} \frac{12x(4-y)}{43}; & 1-y < x < 2-y, 0 < y < 1 \\ 0; & \text{else where} \end{cases} \qquad (2)$$

We can display the geometric form of the domain of the function as follows:

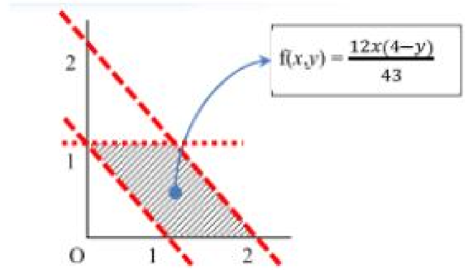

Figure 2.The domain of $f(x,y) = \frac{12x(4-y)}{43}$

By using maple, we obtain the mean of X (51/43), mean of Y (17/43), variance of X (1271/9245), variance of Y (1367/18490), covariance of X & Y (-1868/9245), and correlation of X & Y (-0.668). This result is in accordance with the established theory, as produced in Example 1.

3. METHODOLOGY

In this study, there are three steps undertaken as follows:

a. Constructing functions not satisfying the non-negative property.
b. Computing some statistic measures.
c. Examining problems resulting from neglecting the non-negative properties.

4. RESULTS AND DISCUSSION

Referring to Example 2, where the non-negative property is satisfied. By just changing the bound of Y, as new function not satisfying the non-negative property is obtained. In this case, the interval 0<Y<1 is changed into 0<Y<2, such that X will have negative values.

Example 3. The joint probability density function of the random variables X and Y is

$$f(x,y) = \begin{cases} \frac{3x(4-y)}{11}; & 1-y < x < 2-y, 0 < y < 1 \\ 0; & \text{else where} \end{cases} \quad (3)$$

We can display the geometric form of the domain of the function as follows:

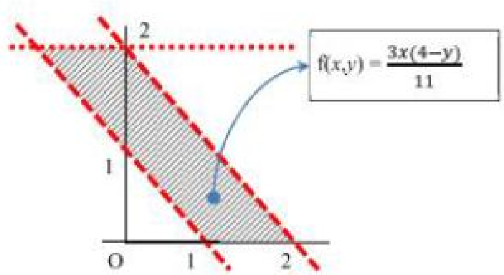

Figure 3.The domain of $f(x,y) = 3x(4-y)/11$

The geometric illustrations above shows that a part of the domain contains X<0, which will result in negative values of the function. Thus, the non-negative property is no longer satisfy by the function f(x,y) = 3x(4 – y)/11.

By using maple, we obtain:

```
>b:=int(int(f,x=1-y..2-y),y=0..2); the result is b equal to 1 (one).
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It means that the function satisfies the probability value property for pdf. The statistic measures gives the value of mean of X (14/11), mean of Y (4/11), variance of X (-123/1210), variance of Y (-14/605), covariance of X & Y (172/1815), and correlation of X & Y (1.9539).

Consequence 01

If the non-negative property is neglected, then the variances could be negative

Consequence 02

If the non-negative property is neglected, then the correlation coefficient could be greater than 1.

Example 4. The joint probability density function of the random variables X and Y is

$$f(x,y) = \begin{cases} \dfrac{24x(1-y)}{13}; & 1-y < x < 2-y, 0 < y < 2 \\ 0; & \text{else where} \end{cases}$$

We can display the geometric form of the domain of the function as follows:

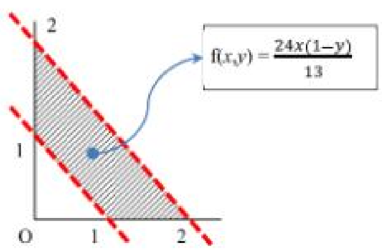

Figure 4.The domain of f(x,y) =24x(1 – y)/13

The geometric illustrations above shows that the part of the domain lies in Quadrant I, which means that X > 0 and Y > 0. However, 0 < Y < 2 will give negative value for (1 – Y), especially for Y > 1. Thus, the non-negative property is no longer satisfy by the function f(x,y) = 24x(1–y)/13.

By using maple, we obtain:

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>b:=int(int(f,x=1-y..2-y),y=0..1)+int(int(f,x=0..2-y),y=1..2); the result is b equal to 1 (one).

It means that the function satisfies the probability value property for pdf. The statistic measures gives the value of mean of X (88/65), mean of Y (1/5), variance of X (251/4225), variance of Y (-18/325), covariance of X & Y (12/325), and correlation of X & Y (-0.64 *I*) whereas I is an imaginary number.

Consequence 03.

If the non-negative property is neglected, then the correlation coefficient could be an imaginary number.

Example 5. Referring to Example 3, find the probability value for P(1–*y*<x<2–*y*,1½<*y*< 2) and obtain:

>b:=int(int(f,x=1-y..2-y),y=3/2..2)); $p := \frac{-13}{176}$ (*negative probability value*).

Consequence 04.

If the non-negative property is neglected, then the probability value could be negative.

5. CONCLUSION

There are four possibilities when the non-negative property is neglected, namely: the variances could be negative, the correlation coefficient could be greater than 1, the correlation could be imaginary, and the probability value could be negative.

In mathematics, the history of an imaginary number $\sqrt{-1}$ is very interesting, where a non-existing number is investigated and we find many advantages of it. In addition, the membership of a set must be well-defined, but nowadays, the theory of fuzzy set does not need a membership definition like in classic set theory. In the theory of probability, it is possible to explore more on the property of probability density function.

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