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### SIR Model Analysis for Transmission of Dengue Fever Disease with Climate Factors Using Lyapunov Function

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Abstract: The aim of this paper is to discuss the dynamic system of SIR model with climate factors for transmission characteristics of dengue fever in closed population. Human population is divided into three types, which are susceptible population, infected population, and recovered population, while mosquito population is divided into two kinds that are susceptible population and infected population contaminated by dengue fever virus. Five nonlinear differential equations are analyzed to obtain the equilibrium of the system and the basic reproduction number  $R_0$ . Next, the stability of the equilibrium of the system is dissected with three theorems using lyapunov. The results will be in the form of the disease-free equilibrium, the endemic equilibrium, and the basic reproduction number. The basic reproduction number  $R_0$  is influenced by climate factors. If  $R_0 \leq 1$ , the disease-free equilibrium will be asymptotically stable, whereas if  $R_0 > 1$ , the endemic equilibrium will become asymptotically stable instead. Based on the results, the dengue fever transmission is affected greatly by the climate of a region.

#### 1. Introduction

Infectious diseases were estimated to vanish along with the development of medical research for vaccines, antibiotics, and life quality enhancement in the middle of 20th century. However, in fact, in the beginning of 21st century, infectious diseases still caused sorrow in the developing countries. Malaria, jaundice, AIDS, and Ebola are some of the culprits. Among all of those diseases, one of the easily transferred diseases is dengue fever. Dengue fever spread in the Southeast Asia and all over the world, particularly in the tropical climate countries [1,2]

Some articles have discussed the dengue fever transmission model, SIR model, and looked for the disease-free equilibrium and the three endemic equilibriums to reduce the sufferers of dengue fever [4], modelling dengue fever by doing detection and prediction of dengue fever in Semarang [3], simulating dengue fever transmission in Selangor (Malaysia) [5], modelling dengue fever transmission by paying attention on the parameter of the mobility of citizens between two cities [7,8,9]. In this article, the mathematics modelling for dengue fever transmission by including climate factor as an influential parameter toward model, disease-free equilibrium and endemic equilibrium analyzed by using Lyapunov function following steps [6,13] that discovered the global equilibrium for SIR and SEIR mathematics model in dengue fever transmission.



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#### 2. Formulation Of Sir Model With Climate Factors

Human population is divided into three classes, Susceptible, Infected, and Recovery (Immune), while mosquito population is divided into two classes - Susceptible and Infected. At time *t*, we denote the susceptible humans for dengue fever, the infectious humans, the recovered humans in the particular range of time, susceptible mosquitos for dengue fever, and the infectious mosquitos as  $S_H$ ,  $I_H$ ,  $R_H$ ,  $S_V$ , and  $I_V$  respectively. No immune class exists for mosquitos because the infected period ends when the mosquitos die.  $N_H = S_H + I_H + R_H$  and  $N_V = S_V + I_V = \frac{A}{\mu_V}$  are the sum of human population and

vector at time t.  $\mu_H$  is birth and death rate for humans and  $\mu_V$  is for mosquitos.

The alteration in each groups of humans and mosquitos with climate factor  $\alpha$  can be interpreted into Figure 1 as follows:



Figure 1. Scheme of human population and vector for SIR model.

Scheme of human population and vector in Figure 1 can be shown into mathematics models that form nonlinear differential equations as seen in equation (1). From those models, the disease-free equilibrium and the endemic equilibrium will be obtained by using Lyapunov function.

$$\begin{cases} S_{H}^{'} = \mu_{H} N_{H} - \left(\frac{\beta_{H} b + \alpha}{N_{H}}\right) S_{H} I_{V} - \mu_{H} S_{H} \\ I_{H}^{'} = \left(\frac{\beta_{H} b + \alpha}{N_{H}}\right) S_{H} I_{V} - (\gamma_{H} + \mu_{H}) I_{H} \\ R_{H}^{'} = \gamma_{H} I_{H} - \mu_{H} R_{H} \\ S_{V}^{'} = \mu_{V} N_{V} - \left(\frac{\beta_{V} b + \alpha}{N_{H}}\right) S_{V} I_{H} - \mu_{V} S_{V} \\ I_{V}^{'} = \left(\frac{\beta_{V} b + \alpha}{N_{H}}\right) S_{V} I_{H} - \mu_{V} I_{V} \end{cases}$$
(1)

Since  $N_V = S_V + I_V = \frac{A}{\mu_V}$ , equation (1) can be simplified as that in equation (2)

$$\begin{split} S_{H}^{'} &= \mu_{H} N_{H} - \left(\frac{\beta_{H} b + \alpha}{N_{H}}\right) S_{H} I_{V} - \mu_{H} S_{H} \\ I_{H}^{'} &= \left(\frac{\beta_{H} b + \alpha}{N_{H}}\right) S_{H} I_{V} - (\gamma_{H} + \mu_{H}) I_{H} \\ R_{H}^{'} &= \gamma_{H} I_{H} - \mu_{H} R_{H} \\ S_{V}^{'} &= A - \left(\frac{\beta_{V} b + \alpha}{N_{H}}\right) S_{V} I_{H} - \mu_{V} S_{V} \\ I_{V}^{'} &= \left(\frac{\beta_{V} b + \alpha}{N_{H}}\right) S_{V} I_{H} - \mu_{V} I_{V} \end{split}$$

(2)

(4)

All of the parameters and the parameters of the models are non-negative. They are clearly shown in the system (2), the non-negatif octant  $R_{+}^{5}$  is positively invariant. Related to the system (2), we can obtain the following theorems.

**Theorem 1.** Let  $(S_H(t)>0, I_H(t)>0, R_H(t)>0, S_V(t)>0, I_V(t)>0)$  be the solution of the system (2) with the initial condition  $(S_{0H}, I_{0H}, R_{0H}, S_{0V}, I_{0V})$  and set D

$$D = \left\{ \left( S_{H}, I_{H}(t), R_{H}(t), S_{V}, I_{V} \right) \in R_{*}^{s}, F_{1} \le N_{H}, F_{2} \le \frac{A}{\mu_{V}} \right\}$$
(3)

for model (2), *D* is a positively invariant set covering all of the solutions in  $R_+^5$ . **Proof.** Consider the function below be the future Lyapunov function:

$$F(t) = (F_1(t), F_2(t)) = (S_H + I_H + R_H, S_V + I_V)$$

Next, F(t) will be derived according to time (t)

$$\begin{split} F'(t) &= \left(F_{1}'(t), F_{2}'(t)\right) = \left(S_{H} + I_{H} + R_{H}, S_{V} + I_{V}\right) \\ &= \left(\mu_{H} N_{H} - \left(\frac{\beta_{s}b + \alpha}{N_{s}}\right) S_{H} I_{V} - \mu_{H} S_{H} + \left(\frac{\beta_{H}b + \alpha}{N_{H}}\right) S_{H} I_{V} - (\gamma_{H} + \mu_{H}) I_{H} + \gamma_{H} I_{H} - \mu_{H} R_{H}, \\ A - \left(\frac{\beta_{s}b + \alpha}{N_{s}}\right) S_{V} I_{H} - \mu_{V} S_{V} + \left(\frac{\beta_{s}b + \alpha}{N_{s}}\right) S_{V} I_{H} - \mu_{V} I_{V} \\ &= \left(\mu_{H} N_{H} - \mu_{H} \left(S_{H} + I_{H} + R_{H}\right), A - \mu_{V} \left(S_{V} + I_{V}\right)\right) \\ &= \left(\mu_{H} N_{H} - \mu_{H} F_{1}, A - \mu_{V} F_{2}\right) \end{split}$$

Therefore, we get

 $\begin{cases} F_1'(t) = \mu_H N_H - \mu_H F_1 \le 0 \\ F_2'(t) = A - \mu_V F_{21} \le 0 \end{cases}$ 

From equation (4),  $F'(t) \le 0$  indicating D is a positively invariant set. By solving equation (4), we get

$$0 \le (F_1(t), F_2(t)) \le \left(N_H + F_1(0)e^{-\mu_H t}, \frac{A}{\mu_V} + F_2(0)e^{\mu_V t}\right)$$

where  $F_1(0)$  and  $F_2(0)$  are the values of  $F_1(t)$  and  $F_2(t)$ .

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From  $t \longrightarrow \infty, 0 \le (F_1(t), F_2(t)) \le \left(N_H, \frac{A}{\mu_V}\right)$ , we can conclude that *D* is a positively invariant set. This

theorem guarantees that in a place, where in the beginning no virus bearer of dengue fever exists, turns into a contaminated place after the susceptible population, the not infected yet population  $(S_{H})$ , the infected population  $(I_H)$ , and the recovered population  $(R_H)$  were found.

#### 3. Sir Model Stability Analysis

By applying Diekhmann and Heesterbeek method [10, 11], Basic Reproduction can be obtained from system (2)

$$R_{0} = \frac{A(b(b\beta_{H}\beta_{V} + \beta_{H}\alpha + \beta_{V}\alpha) + \alpha^{2})}{N_{V}\mu_{V}(\mu_{H} + \gamma_{H})}$$
(5)

System (2) has two equilibriums:  $R_0 \le 1$  of which the only equilibrium is the disease-free

 $K^{*} = (S_{H}^{*}, I_{H}^{*}, R_{H}^{*}, S_{V}^{*}, I_{H}^{*}) = \left(N_{H}, 0, 0, \frac{A}{\mu_{V}}, 0\right) \in D$ equilibrium can be obtaiged,  $K^{**} = (S_{H}^{**}, I_{H}^{**}, R_{H}^{**}, S_{V}^{**}, I_{H}^{**}) \in D$  where  $S_{H}^{**}, I_{H}^{**}, R_{H}^{**}, S_{V}^{**}, I_{H}^{**} > 0$  with

$$S_{n}^{"} = \frac{\mu_{v} N_{n}^{2} (b\beta_{v} \mu_{u} + \mu_{u} \mu_{v} + \mu_{u} \alpha + \mu_{v} \gamma_{u})}{(b\beta_{v} + a) (Ab\beta_{u} + N_{n} \mu_{u} \mu_{v} + A\alpha)}$$

$$I_{H}^{"} = \frac{N_{n} \mu_{u} (Ab^{2} \beta_{v} \mu_{u} + Ab\beta_{u} \alpha + Ab\beta_{v} \alpha - N_{n} \mu_{u} \mu_{v}^{2} + A\alpha^{2})}{(b\beta_{v} + a) (Ab\beta_{n} \mu_{u} + Ab\beta_{u} \gamma_{u} + N_{u} \mu_{u} \mu_{v} \gamma_{u} + A\mu_{u} \alpha + A\alpha\gamma_{u})}$$

$$R_{n}^{"} = \frac{N_{u} (Ab^{2} \mu_{v} \beta_{u} + Ab\beta_{u} \alpha + Ab\beta_{v} \alpha - N_{u} \mu_{u} \mu_{v}^{2} + A\alpha^{2}) \gamma_{u}}{(b\beta_{v} + a) (Ab\beta_{v} \beta_{u} + Ab\beta_{u} \alpha + Ab\beta_{v} \alpha - N_{u} \mu_{u} \mu_{v} \gamma_{u} + A\mu_{u} \alpha + A\alpha\gamma_{u})}$$

$$S_{v}^{"} = \frac{Al^{2} \mu_{u} (Ab^{2} \beta_{v} \mu_{u} + Ab\beta_{u} \gamma_{u} + N_{u} \mu_{u}^{2} \mu_{v} + N_{u} \mu_{u} \mu_{v} \gamma_{u} + A\mu_{u} \alpha + A\alpha\gamma_{u}}{b^{2} \beta_{v} \beta_{u} \mu_{u} + b\beta_{u} \mu_{u} \mu_{v} + b\beta_{u} \mu_{u} \alpha + b\beta_{u} \mu_{u} \gamma_{u} + A\mu_{u} \alpha + \mu_{u} \alpha^{2} + \mu_{v} \alpha\gamma_{u}}$$

$$I_{v}^{**} = \frac{\mu_{u} (Ab^{2} \beta_{v} \mu_{u} + Ab\beta_{u} \alpha + Ab\beta_{v} \alpha - N_{u} \mu_{u} \mu_{v} \gamma_{u} + A\mu_{u} \alpha + A\alpha\gamma_{u}}{(Ab\beta_{u} \mu_{u} + b\beta_{u} \mu_{u} \mu_{v} + b\beta_{u} \mu_{u} \alpha + b\beta_{u} \mu_{u} \gamma_{u} + A\beta_{v} \alpha + \lambda\alpha\gamma_{u}})$$

$$K_{v}^{**} = \frac{\mu_{u} (Ab^{2} \beta_{v} \mu_{u} + Ab\beta_{u} \alpha + Ab\beta_{v} \alpha - N_{u} \mu_{u} \mu_{v} \gamma_{u} + A\mu_{u} \alpha + A\alpha\gamma_{u}}){(Ab\beta_{u} \mu_{u} + Ab\beta_{u} \gamma_{u} + N_{u} \mu_{u} \mu_{v} + A\alpha^{2} + \mu_{u} \alpha^{2} + \mu_{v} \alpha\gamma_{u}}$$

System (2) always has a disease-free equilibrium  $K^* = (S_H^*, I_H^*, R_H^*, S_V^*, I_H^*) = \left(N_H, 0, 0, \frac{A}{H_V}, 0\right) \in D$ that shows the no dengue fever situation.

**Teorema 2.** If  $R_0 \le 1$ , the disease-free equilibrium  $K^* = (S_H^*, I_H^*, R_H^*, S_V^*, I_H^*) = \left(N_H, 0, 0, \frac{A}{\mu_V}, 0\right) \in D$  is

asymptotically stable in D with assumptions

Prof. Consider the function below be the candidate of Lyapunov function  $G(t) = (S_H - S_H^* \ln S_H) + I_H + R_H + (S_V - S_V^* \ln S_V) + I_V$ 

(8)

Function (8) is derived to time (t), so

$$G'(t) = S'_{_{H}} \left(1 - \frac{S^{*}_{_{H}}}{S_{_{H}}}\right) + I'_{_{H}} + R'_{_{H}} + S'_{_{V}} \left(1 - \frac{S^{*}_{_{V}}}{S_{_{V}}}\right) + I'_{_{V}}$$

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$$= \left( \left( \mu_{H}N_{H} - \left( \frac{\beta_{v}b + \alpha}{N_{h}} \right) S_{H}I_{v} - \mu_{H}S_{H} \right) \left( 1 - \frac{S_{u}^{*}}{S_{H}} \right) \right) + \left( \left( \frac{\beta_{h}b + \alpha}{N_{H}} \right) S_{H}I_{v} - (\gamma_{H} + \mu_{H})I_{H} \right) = \mu_{H}N_{H} \left( 1 - \frac{S_{u}^{*}}{S_{H}} \right) + I_{v} \left( \left( \frac{\beta_{v}b + \alpha}{N_{h}} \right) S_{v}^{*} - \mu_{V} \right) + I_{H} \left( \frac{\beta_{v}b + \alpha}{N_{h}} \right) S_{v}^{*} - \mu_{H} \right) + \mu_{H}S_{H}^{*} \left( 1 - \frac{S_{u}^{*}}{S_{u}^{*}} \right)$$

$$+ \left( \gamma_{H}I_{H} - \mu_{H}R_{H} \right) + \left( \left( A - \left( \frac{\beta_{v}b + \alpha}{N_{h}} \right) S_{v}I_{H} - \mu_{v}S_{v} \right) \right) + \left( \left( \frac{\beta_{v}b + \alpha}{N_{h}} \right) S_{v}I_{H} - \mu_{v}I_{v} \right) + A \left( 1 - \frac{S_{v}^{*}}{S_{v}} \right) + \mu_{v}S_{v}^{*} \left( 1 - \frac{S_{v}}{S_{v}^{*}} \right) - \mu_{H}R_{H}$$
By using assumption in (8), we get
$$G'(t) = \mu_{H}N_{H} \left( 1 - \frac{S_{H}^{*}}{S_{H}} \right) + \mu_{H}S_{H}^{*} \left( 1 - \frac{S_{v}^{*}}{S_{v}^{*}} \right) - \mu_{H}R_{H}$$

$$(10)$$

Based on the disease-free equilibrium, we have  $S_H^* = N_H$  and  $S_V^* = \frac{A}{\mu_V}$ , then equation (10) can be

$$\begin{aligned} G'(t) &= \mu_{H} N_{H} \left( 1 - \frac{S_{H}^{*}}{S_{H}} \right) + + \mu_{H} N_{H} \left( 1 - \frac{S_{V}}{S_{V}^{*}} \right) + A \left( 1 - \frac{S_{V}^{*}}{S_{V}} \right) - \mu_{H} R_{H} \\ &= \mu_{H} N_{H} \left( 2 - \frac{S_{H}^{*}}{S_{H}} - \frac{S_{H}}{S_{H}^{*}} \right) + A \left( 2 - \frac{S_{V}^{*}}{S_{V}} - \frac{S_{V}}{S_{V}^{*}} \right) - \mu_{H} R_{H} \\ G'(t) &= -\mu_{H} N_{H} \frac{\left( S_{H} - S_{H}^{*} \right)^{2}}{S_{H} \frac{S_{H}}{S_{H}}} - A \frac{\left( S_{V} - S_{V}^{*} \right)^{2}}{S_{V} S_{V}^{*}} - \mu_{H} R_{H} \end{aligned}$$
(11)

The result is  $G'(t) \leq 0$ . By using Lasalle's Extension to Lyapunov's method [13], the compact set for each solution is in the largest invariant set with  $S_H = S_H^*$ ,  $S_V = S_V^*$  and  $R_H = 0$  is singleton  $\{P^*\}$ . However, it can be concluded that the disease-free equilibrium,  $P^*$ , is asymptotically stable in D which becomes the proof of the theorem.

#### 4. The Stability of Endemic Equilibrium

System (2) owns some endemic equilibriums  $K^{**}$  that indicate the existence of dengue fever sufferer. **Theorem 3.** If  $R_0 \le 1$ , the endemic equilibrium  $K^{**}$  is asymptotically stable in D with assumptions as follow

$$\begin{cases} N_{H} = S_{H}^{**} \\ \mu_{H} = \frac{A}{S_{V}^{**}} \end{cases}$$
(12)

$$b = \frac{A(\gamma_H + \mu_H) - \alpha p (S_V^{**})^2}{p \beta_V (S_V^{**})^2}$$
with

$$\begin{cases} p = \frac{\beta_H b + \alpha}{N_H} \\ q = \frac{\beta_V b + \alpha}{N_H} \end{cases}$$
(13)

Proof. Consider the function below be the candidate of Lyapunov function

$$W(t) = (S_{H} - S_{H}^{*} \ln S_{H}) + I_{H} + \left(\frac{\gamma_{H} + \mu_{H}}{q S_{V}^{**}}\right) (S_{V} - S_{V}^{*} \ln S_{V}) + \left(\frac{\gamma_{H} + \mu_{H}}{q S_{V}^{**}}\right) I_{V}$$
(14)

Function (14) is derived according to time (t), so we can get

$$\begin{split} W'(t) &= S'_{\mu} \left( 1 - \frac{S'_{\mu}}{S_{\mu}} \right) + I'_{\mu} + \left( \frac{\gamma_{\mu} + \mu_{\mu}}{qS_{v}^{v+}} \right) S'_{v} \left( 1 - \frac{S'_{v}}{S_{v}} \right) + \left( \frac{\gamma_{\mu} + \mu_{\mu}}{qS_{v}^{v+}} \right) I'_{v} \\ &= \left( \left( \mu_{\mu}N_{\mu} - \left( \frac{\beta_{\nu}b + \alpha}{N_{\pi}} \right) S_{\mu}I_{\nu} - \mu_{\mu}S_{\mu} \right) \left( 1 - \frac{S'_{\mu}}{S_{\mu}} \right) \right) + \left( \left( \frac{\beta_{\mu}b + \alpha}{N_{\mu}} \right) S_{\mu}I_{\nu} - (\gamma_{\mu} + \mu_{\mu})I_{\mu} \right) \\ &+ \left( \frac{\gamma_{\mu} + \mu_{\mu}}{qS_{v}^{v+}} \right) \left( \left( A - \left( \frac{\beta_{\nu}b + \alpha}{N_{\pi}} \right) S_{\nu}I_{\mu} - \mu_{\nu}S_{\nu} \right) \left( 1 - \frac{S'_{\nu}}{S_{\nu}} \right) \right) \\ &+ \left( \frac{\gamma_{\mu} + \mu_{\mu}}{qS_{v}^{v+}} \right) \left( \left( \frac{\beta_{\nu}b + \alpha}{N_{\pi}} \right) S_{\nu}I_{\mu} - \mu_{\nu}I_{\nu} \right) \end{split}$$

$$= \left(\mu_{H}N_{H}\left(1-\frac{S_{H}}{S_{H}}\right) + \mu_{H}S_{H}^{*}\left(1-\frac{S_{H}}{S_{H}^{*}}\right)\right) + I_{V}\left(\left(\frac{\beta_{L}b+\alpha}{N_{s}}\right)S_{V}^{*} - \left(\frac{\gamma_{H}+\mu_{H}}{qS_{V}^{**}}\right)\mu_{V}\right)$$

$$+ I_{H}\left(\left(\frac{\gamma_{H}+\mu_{H}}{qS_{V}^{**}}\right)\left(\frac{\beta_{L}b+\alpha}{N_{s}}\right)S_{V}^{*} - \mu_{H}\right) + \left(\frac{\gamma_{H}+\mu_{H}}{qS_{V}^{**}}\right)A\left(\left(1-\frac{S_{V}}{S_{V}}\right) + \left(1-\frac{S_{V}}{S_{V}^{*}}\right)\right)$$
(15)

By using assumption 12, equation (15) becomes  $W'(t) = \left(\mu_{H}N_{H}\left(1 - \frac{S_{H}}{S_{H}}\right) + \mu_{H}N_{H}\left(1 - \frac{S_{H}}{S_{H}}\right)\right) + \mu_{v}\left(\frac{\gamma_{H} + \mu_{H}}{q}\right) \left(\left(1 - \frac{S_{v}}{S_{v}}\right) + \left(1 - \frac{S_{v}}{S_{v}}\right)\right) \\
= \left(\mu_{H}N_{H}\left(2 - \frac{S_{H}^{*}}{S_{H}} - \frac{S_{H}}{S_{H}^{*}}\right)\right) + \mu_{v}\left(\frac{\gamma_{H} + \mu_{H}}{q}\right) \left(2 - \frac{S_{v}^{*}}{S_{v}} - \frac{S_{v}}{S_{v}^{*}}\right) \\
W'(t) = -\mu_{H}N_{H}\left(\frac{\left(S_{H} - S_{H}^{*}\right)^{2}}{S_{H}S_{H}^{*}}\right) - \mu_{v}\left(\frac{\gamma_{H} + \mu_{H}}{q}\right) \left(\frac{\left(S_{v} - S_{v}^{*}\right)^{2}}{S_{v}S_{v}^{*}}\right)$ (16)

Condition (16) guarantees  $W(t) \le 0$  for all  $(S_H, I_H, R_H, S_V, I_V) \in D$  and W(t) = 0 if and only if  $S_H = S_H^*, I_H = I_H^*, R_H = R_H^*, S_V = S_V^*$  and  $I_V = I_V^*$ . It makes point  $P^{**}$  becoming the only positively invariant set in system (2) that fully exists in  $L = \{(S_H, I_H, R_H, S_V, I_V), S_H = S_H^*, I_H = I_H^*, R_H = R_H^*, S_V = S_V^*, I_V = I_V^*\}$  and based on the asymptotically stable theorem [12], the endemic equilibrium  $P^{**}$  satisfies the theorem in D. From this fact, Theorem 3 has proven.

#### 5. Conclusion

There are three theorems obtained in this article about the existence of dengue fever transmission, the disease-free status, and the endemic status of dengue fever transmission. The stability analysis of the disease-free equilibrium and the endemic equilibrium is undertaken by using lyapunov function. If  $R_0 \leq 1$ , the disease-free equilibrium is asymptotically stable, then the disease disappears from the population means that no one suffers from dengue fever. Meanwhile, if  $R_0 > 1$ , the endemic equilibrium becomes asymptotically stable, which means the infected human will contaminate the disease to at least one individual in the population.

#### References

- Fourth Intergovernmental Preparatory Meeting, Health and the environment in the WHO European Region: Situation and Policy at the beginning of the 21st Century, WHO Regional Office for Europe, Copenhagen, 2004.
- [2] C.J.L. Murray, A.D. Lopez (Eds.), The global burden of disease: a comprehensive assessment of mortality and disability from diseases, injuries and risk factors in 1990 and projected to 2020, Global Burden of Disease and Injury, vol. 1, Harvard School of Public Health on behalf of WHO, Cambridge, MA, 1996.
- [3] Derouich, M. dan Boutayeb, A.. Dengue fever: Mathematical modelling and computer simulation. Applied Mathematics and Computation. 177(2): 528-544. 2006.
- [4] N. Nuraini, E. Soewono dan KA Sidarto. Mathematical Model of Dengue Disease Transmission with Severe DHF Compartment. BULLETIN of the Malaysian Mathematical Sciences Society. 30(2):143-157.2007.
- [5] Side, S., Noorani, S.M., 2013. A SIR Model for Spread of Dengue Fever Disease (Simulation for South Sulawesi, Indonesia and Selangor, Malaysia). World Journal of Modelling and Simulation. 9(2):96-105.
- [6] Syafruddin, S., Noorani, M.S.Md., 2013. Lyapunov function of SIR and SEIR model for transmission of dengue fever disease. *International Journal of Simulation and Process Modelling*. 8(2/3), 177-184.
- [7] Rangkuti, Y.M., Side, S., Noorani, M.S.M., 2014. Numerical analytic solution of SIR model of dengue fever diseasein South Sulawesi using homotopy perturbation method and variational iteration method. *Journal of Mathematical and Fundamental Sciences*.46A(1). 91-105

- [8] Chen Y, Yan M. dan Xiang Z. Transmission Dynamics of a Two-City SIR Epidemic Model with Transport-Related Infections. Journal of Applied Mathematics. 2014.
- [9] Constantina AW, Yessica N, dan Hendry. Deteksi dan Prediksi Daerah Endemis Demam Berdarah Dengue (DBD) dengan Pemodelan Matematis Susceptible, Infected, Recovered (SIR) (Studi Kasus : Kabupaten Semarang). Jurnal Teknologi Informasi-Aiti, 8(2) : 101 – 200. 2011.
- [10] Diekmaan, O., Heesterbeek, J.A. and Metz, J.A.J. On the definition and computation of the basic reproduction ratio R infectious disease in heterogenous population. J. Math. And Biol., 28(4): 365-382. 1990.
- [11] Diekmaan, O., Heesterbeek, J.A. and Metz, J.A.J. Mathematical epidemology of infectious disease, model building, analysis and interpretation. Wiley series in mathematical and computational biology. John Wiley & Sons Ltd., Chicester. 2000.
- [12] LaSalle JP. The stability of dynamical systems. Philadelphia: SIAM. 1976.
- [13] Tewa, J.J., Dimi, J.L. dan Bowong, S.. Lyapunov function for a dengue disease transmission model. Chaos, Solutions and Fractals. 39(2):936-941. 2009

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