An Over view on intuitionistic fuzzy topological spaces To

by Muhammad Abdy

Journal of Physics: Conference Series

PAPER • OPEN ACCESS

An Over view on intuitionistic fuzzy topological spaces

An Over view on intuitionistic fuzzy topological spaces

M Abdy[1*], S Zenin[2] and Irwan[1]

[1]Department of Mathematics, FMIPA, Universitas Negeri Makassar, Indonesia
[2]Department of Mathematics with Computer Graphics, Faculty of Science and Natural Resources, Universiti Malaysia Sabah, Malaysia

*e-mail: muh.abdy@unm.ac.id

Abstract. We present a brief overview some fundamental results on the intuitionistic fuzzy topological spaces, and give some introductory results about fuzzy open set, fuzzy closed set, fuzzy neighborhood, fuzzy interior set, fuzzy continuity, fuzzy compactness and fuzzy connectedness in these spaces.

Keywords: Fuzzy sets, instuitionistic fuzzy sets, topological, spaces

1. Introduction

The theory of fuzzy sets proposed by Zadeh [1] in 1965, has shown successful applications in various fields. After the pioneering work of Zadeh, some researchers began to study both the theory and its applications. Chang [2] defined fuzzy topology by utilizing the definition of topology in the classical sets. Then [3] and [4] introduced fuzzy graphs and fuzzy groups. Furthermore, several other researchers continue to develop the theoretical aspects of the fuzzy set [6][7][8][9][10][11].

In the fuzzy set theory, the membership degree of an element is a value at [0, 1]. However, it may not always be true that the nonmembership degree of an element in the fuzzy set is equal to 1 minus the membership degree because there may be some degree of hesitation in determining the membership degree. Therefore, a generalization of the fuzzy sets was introduced by Atanassov [12], [13], known as intuitionistic fuzzy sets. Some applications of the Atanassov's concept have been successfully implemented, such as in the decision making, medical field, pattern recognition, and so on [14].

An important problem in intuitionistic fuzzy sets is to obtain an appropriate concept of intuitionistic fuzzy topological spaces. The problem has been studied by Coker [15]. He has defined the notion of intuitionistic fuzzy topological spaces refer to Chang's topology concept. The concept of (r,s)-connected fuzzy sets in intuitionistic fuzzy topological spaces was introduced [16] and investigated some properties of them. Then [17] presented the notion of intuitionistic fuzzy points and fuzzy neighborhoods, and [18] studied some types of fuzzy connectedness in Coker's intuitionistic fuzzy topological spaces concept. Park [19] introduced the intuitionistic fuzzy metric spaces concept. Recently, [20] investigated the concept of intuitionistic I-fuzzy quasicoincident neighborhood systems of intuitionistic fuzzy points. They investigated the relation between the category of intuitionistic I-fuzzy quasicoincident neighborhood spaces and the category of intuitionistic I-fuzzy topological spaces, and construct the concept of generated intuitionistic I-fuzzy topology by using fuzzifying topologies. The main purpose of this paper is to overview of the concepts of topology in intuitionistic fuzzy sets, such as intuitionistic fuzzy open set, intuitionistic fuzzy closed set, intuitionistic fuzzy neighborhood, intuitionistic fuzzy interior set, intuitionistic fuzzy continuity, intuitionistic fuzzy compactness, and intuitionistic fuzzy connectedness.

2. Brief introduction of intuitionistic fuzzy set

Definition 2.1.

Let U be a nonempty fixed set. An intuitionistic fuzzy set (IFS in short) A, written as $\hat{A}$, in U is a set having the form

$$\hat{A}=\{(x,\mu_A(x),\gamma_A(x))\mid x\in U\}$$

where the value of the functions $\mu_{\hat{A}}:U\to[0,1]$ and $\gamma_{\hat{A}}:U\to[0,1]$ define the membership degree and non-membership degree of each element $x\in U$ to the set $\hat{A}$, respectively, and $\forall x\in U$, we have

$$0\le\mu_{\hat{A}}(x)+\gamma_{\hat{A}}(x)\le 1$$

The amount $\eta_{\hat{A}}(x)=1-(\mu_{\hat{A}}(x)+\gamma_{\hat{A}}(x))$ is called the hesitation part, which may cater to either membership value or nonmembership value or both. For the sake simplicity, we shall use the symbol $\hat{A}=(x,\mu_{\hat{A}},\gamma_{\hat{A}})$ for the IFS $\hat{A}=\{(x,\mu_{\hat{A}}(x),\gamma_{\hat{A}}(x))\mid x\in U\}$

Example 2.2.

Let $A=\{(x,\mu_A(x))\mid x\in U\}$ be a fuzzy set on a nonempty set U. We can denote the fuzzy set A as $A=\{(x,\mu_A(x),1-\mu_A(x))\mid x\in U\}$. It's obviously that every fuzzy set A on U is an IFS.

Definition 2.2.

Let $\hat{A}=(x,\mu_{\hat{A}},\gamma_{\hat{A}})$ and $\hat{B}=(x,\mu_{\hat{B}},\gamma_{\hat{B}})$ be two IFSs in nonempty set U, then:

1. $\hat{A}\subseteq\hat{B}$ iff $\mu_{\hat{A}}\le\mu_{\hat{B}}$ and $\gamma_{\hat{A}}\ge\gamma_{\hat{B}}$ $\forall x\in U$
2. $\hat{A}=\hat{B}$ iff $\hat{A}\subseteq\hat{B}$ and $\hat{A}\supseteq\hat{B}$
3. $\overline{\hat{A}}=(x,\gamma_{\hat{A}},\mu_{\hat{A}})$
4. $\hat{A}\cup\hat{B}$ = $(x,\max(\mu_{\hat{A}},\mu_{\hat{B}})),\min(\gamma_{\hat{A}},\gamma_{\hat{B}}))$.
5. $\hat{A}\cap\hat{B}$ = $(x,\min(\mu_{\hat{A}},\mu_{\hat{B}})),\max(\gamma_{\hat{A}},\gamma_{\hat{B}}))$ …
6. $[]\hat{A}$ = $(x,\mu_{\hat{A}},1-\mu_{\hat{A}})$
7. $\langle\rangle\hat{A}$= $(x,1-\gamma_{\hat{A}},\gamma_{\hat{A}})$

Coker [15] generalized the operations of intersection and union in Definition 2.2 to any collections of IFSs as follows

Definition 2.3.

Let $\{\hat{A}_i\mid i\in J\}$ be an arbitrary collections of IFS in U, then

1. $\bigcup_i\hat{A}_i$= $(x,\max_i(\mu_{\hat{A}_i}),\min_i(\gamma_{\hat{A}_i}))$
2. $\bigcap_i\hat{A}_i$= $(x,\min_i(\mu_{\hat{A}_i}),\max_i(\gamma_{\hat{A}_i}))$

Definition 2.4.

Let $1_{\sim}$ and $0_{\sim}$ be IFSs in U, we define as $1_{\sim}=\{(x,1,0)\mid x\in U\}$ .and $0_{\sim}=\{(x,0,1)\mid x\in U\}$.

Corollary 2.5.

Let $\hat{A}$, $\hat{B}$, $\hat{C}$ and $\hat{D}$ be IFSs in U, then:

1. $\hat{A}\subseteq\hat{B}$ and $\hat{C}\subseteq\hat{D}$ then $\hat{A}\cup\hat{C}\subseteq\hat{B}\cup\hat{D}$ and $\hat{A}\cap\hat{C}\subseteq\hat{B}\cap\hat{D}$
2. $\hat{A}\subseteq\hat{B}$ and $\hat{A}\subseteq\hat{C}$ then $\hat{A}\subseteq\hat{B}\cap\hat{C}$
3. $\hat{A}\subseteq\hat{C}$ and $\hat{B}\subseteq\hat{C}$ then $\hat{A}\cup\hat{B}\subseteq\hat{C}$
4. $\hat{A}\subseteq\hat{B}$ and $\hat{B}\subseteq\hat{C}$ then $\hat{A}\subseteq\hat{C}$

5. $\overline{\hat{A}\cup\hat{B}} = \overline{\hat{A}}\cap\overline{\hat{B}}$
6. $\overline{\hat{A}\cap\hat{B}} = \overline{\hat{A}}\cup\overline{\hat{B}}$
7. $\hat{A}\subseteq\hat{B}$ then $\overline{\hat{A}}\supseteq\overline{\hat{B}}$
8. $\overline{\overline{\hat{A}}} = \hat{A}$
9. $\overline{0_{\Box}} = 1_{\Box}$ and $\overline{1_{\Box}} = 0_{\Box}$

Proof.
We will only prove part 6, the others are obviously.
6. Let $\hat{A}=(x,\mu_{\hat{A}},\gamma_{\hat{A}})$, $\hat{B}=(x,\mu_{\hat{B}},\gamma_{\hat{B}})$, then $\hat{A}\cap\hat{B} = (x,\min(\mu_{\hat{A}},\mu_{\hat{B}}),\max(\gamma_{\hat{A}},\gamma_{\hat{B}}))$, so we have
$\overline{\hat{A}\cap\hat{B}} = (x,\max(\gamma_{\hat{A}},\gamma_{\hat{B}}),\min(\mu_{\hat{A}},\mu_{\hat{B}}))$.
And
$\overline{\hat{A}}=(x,\gamma_{\hat{A}},\mu_{\hat{A}})$, $\overline{\hat{B}}=(x,\gamma_{\hat{B}},\mu_{\hat{B}})$, so we have $\overline{\hat{A}}\cup\overline{\hat{B}} = (x,\max(\gamma_{\hat{A}},\gamma_{\hat{B}}),\min(\mu_{\hat{A}},\mu_{\hat{B}}))$
Hence, $\overline{\hat{A}\cap\hat{B}} = \overline{\hat{A}}\cup\overline{\hat{B}}$

Definition 2.6.
Consider U and V two nonempty sets and given a function $f:U\to V$.
(a). Let $\hat{B}=(y,\mu_{\hat{B}},\gamma_{\hat{B}})$ be an IFS in V, the preimage of $\hat{B}$ by f denoted by $f^{-1}(\hat{B})$ is an IFS in U such that $f^{-1}(\hat{B})=\{(x,f^{-1}(\mu_{\hat{B}})(x),f^{-1}(\gamma_{\hat{B}})(x))\mid x\in U\}$
(b). Let $\hat{A}=(x,\mu_{\hat{A}},\gamma_{\hat{A}})$ be an IFS in U, the image of $\hat{A}$ by f denoted by $f(\hat{A})$ is an IFS in V such that $f(\hat{A})=\{(y,f(\mu_{\hat{A}})(y),(1-f(1-\gamma_A))(y))\mid y\in V\}$, with

$$f(\mu_{\hat{A}})=\begin{cases}\sup\limits_{x\in f^{-1}(y)}\mu_{\hat{A}}(x) & ;\ f^{-1}(y)\neq\phi\\ 0 & ;\ f^{-1}(y)=\phi\end{cases}\quad\text{and}$$

$$(1-f(1-\gamma_A))(y))=\begin{cases}\inf\limits_{x\in f^{-1}(y)}\gamma_A(x) & ;\ f^{-1}(y)\neq\phi\\ 1 & ;\ f^{-1}(y)\neq\phi\end{cases}$$

Proposition 2.7. (The properties of images and preimages)
Let $f:U\to V$ be a function, $\hat{A}$ and $\hat{A}_i$ $(i\in J)$ IFSs in U, $\hat{B}$ and $\hat{B}_k$ $(k\in K)$ IFSs in V, with $\hat{A}=(x,\mu_{\hat{A}},\gamma_{\hat{A}})$, $\hat{B}=(y,\mu_{\hat{B}},\gamma_{\hat{B}})$, $\hat{A}_i=(x,\mu_{\hat{A}_i},\gamma_{\hat{A}_i})$, and $\hat{B}_k=(y,\mu_{\hat{B}_k},\gamma_{\hat{B}_k})$,

1. $\hat{A}_1\subseteq\hat{A}_2$ then $f(\hat{A}_1)\subseteq f(\hat{A}_2)$
2. $\hat{B}_1\subseteq\hat{B}_2$ then $f^{-1}(\hat{B}_1)\subseteq f^{-1}(\hat{B}_2)$
3. $\hat{A}\subseteq f^{-1}(f(\hat{A}))$; if f is an injective function then $\hat{A}=f^{-1}(f(\hat{A}))$
4. $f(f^{-1}(\hat{B}))\subseteq\hat{B}$; if f is a surjective function then $\hat{B}=f(f^{-1}(\hat{B}))$
5. $f^{-1}(\bigcup_k\hat{B}_k)=\bigcup_k f^{-1}(\hat{B}_k)$
6. $f^{-1}(\bigcap_k\hat{B}_k)=\bigcap_k f^{-1}(\hat{B}_k)$
7. $f(\bigcup_i\hat{A}_i)=\bigcup_i f(\hat{A}_i)$
8. $f(\bigcap_i\hat{A}_i)\subseteq\bigcap_i f(\hat{A}_i)$; if f is an injective function then $f(\bigcap_i\hat{A}_i)=\bigcap_i f(\hat{A}_i)$
9. $f^{-1}(1_{\Box})=1_{\Box}$; $f^{-1}(0_{\Box})=0_{\Box}$
10. $f(0_{\Box})=0_{\Box}$
11. If f is a surjective function then $f(1_{\Box})=1_{\Box}$

Journal of Physics: Conference Series **1752** (2021) 012005 doi:10.1088/1742-6596/1752/1/012005

12. $f^{-1}(\bar{\hat{B}}) = \overline{f^{-1}(\hat{B})}$

13. If f is a surjective function then $\overline{f(\hat{A})} \subseteq f(\bar{\hat{A}})$

Proof.

6. $f^{-1}(\bigcap_k \hat{B}_k) = f^{-1}(\{(y, \min_k \mu_{\hat{B}_k}, \max_k \gamma_{\hat{B}_k}) \mid y \in V\}) = \{(x, f^{-1}(\min_k \mu_{\hat{B}_k}), f^{-1}(\max_k \gamma_{\hat{B}_k})) \mid x \in U\} =$
$\{(x, \min_k f^{-1}(\mu_{\hat{B}_k}), \max_k f^{-1}(\gamma_{\hat{B}_k})) \mid x \in U\} = \{(x, \min_k(\mu_{f^{-1}(\hat{B}_k)}), \max_k(\gamma_{f^{-1}(\hat{B}_k)})) \mid x \in U\} = \bigcap_k f^{-1}(\hat{B}_k)$

9. $f^{-1}(0_{\sim}) = f^{-1}(\{(y,0,1) \mid y \in V\} = \{(x, f^{-1}(0), f^{-1}(1)) \mid x \in U\} = \{(x,0,1) \mid x \in U\} = 0_{\sim}$

10. $f(0_{\sim}) = \{(y, f(0), (1 - f(1-1)) \mid y \in V\} = \{(y,0,1) \mid y \in V\} = 0_{\sim}$

11. $f(1_{\sim}) = \{(y, f(1), (1 - f(1-0)) \mid y \in V\} = \{(y, f(1), (1 - f(1)) \mid y \in V\}$,
If f is a surjective function then $f(1) = 1$. So that, $\{(y, f(1), (1 - f(1)) \mid y \in V\} =$
$\{(y,1,0) \mid y \in V\} = 1_{\sim}$

12. Since $f^{-1}(\bar{\hat{B}}) = \{(x, f^{-1}(\gamma_{\hat{B}}), f^{-1}(\mu_{\hat{B}})) \mid x \in U\}$ and $\overline{f^{-1}(\hat{B})} = \overline{\{(x, f^{-1}(\mu_{\hat{B}}), f^{-1}(\gamma_{\hat{B}})) \mid x \in U\}} =$ $\{(x, f^{-1}(\gamma_{\hat{B}}), f^{-1}(\mu_{\hat{B}})) \mid x \in U\}$ then we obtain the required result.

3. Intuitionistic fuzzy topological spaces

Coker [15] constructed intuitionistic fuzzy topology or IFT for short concept by generalizing Chang's fuzzy topology concept.

Definition 3.1

Let $\hat{A}$ be an IFS on a nonempty set U and τ is a collection of $\hat{A}$, then τ is said to be IFT for U if it satisfy the following axioms:

(A1) $0_{\sim}, 1_{\sim} \in \tau$

(A2) If $O_1, O_2 \in \tau$ then $O_1 \cap O_2 \in \tau$

(A3) If $O_i \in \tau$ for each $i \in I$ then $\bigcup O_i \in \tau$

The pair (U, τ) is said to be an intuitionistic fuzzy topological spaces (IFTS in short). Any member of τ is called as a τ-intuitionistic fuzzy open set or τ-IFOS for short in U, and the complement of a τ-IFOS in an IFTS is called as τ-intuitionistic fuzzy closed set or τ-IFCS for short.

Proposition 3.2

If (U, τ) is an IFTS on U then several IFTSs on U can be constructed by following way:

1. $\tau_{0.1} = \{[]O \mid O \in \tau\}$
2. $\tau_{0.2} = \{\langle\rangle O \mid O \in \tau\}$

Proof.

We shall only prove 1, and another is similar.

(A1) $1_{\sim} = (x,1,0) = (x,1,1-0) \in \tau_{0.1}$ and $0_{\sim} = (x,0,1) = (x,0,1-0) \in \tau_{0.1}$

(A2) Let $O_1, O_2 \in \tau_{0.1}$, then we have $O_1 = (x, \mu_{O_1}, 1 - \mu_{O_1})$ and $O_2 = (x, \mu_{O_2}, 1 - \mu_{O_2})$. So that $O_1 \cap O_2 = (x, \min(\mu_{O_1}, \mu_{O_2}), \max(1 - \mu_{O_1}, 1 - \mu_{O_2})) = (x, \min(\mu_{O_1}, \mu_{O_2}), 1 - \min(\mu_{O_1}, \mu_{O_2})) \in \tau_{0.1}$

(A3) Let $O_i \in \tau_{0.1}$ then $\bigcup O_i = (x, \max_i \mu_{O_i}, \min_i(1 - \mu_{O_i})) = (x, \max_i \mu_{O_i}, 1 - \max_i(\mu_{O_i})) \in \tau_{0.1}$

Definition 3.3.

Let $\alpha, \beta \in (0,1)$ be two fixed real numbers such that $\alpha + \beta \le 1$ and U is a nonempty set, $x \in U$. Then an IFS on U defined by $p^x_{(\alpha,\beta)} = (x, x_\alpha, 1 - x_{(1-\beta)})$ is called intuitionistic fuzzy point (IFP in short) of U, and x is called the support of $p^x_{(\alpha,\beta)}$.

Journal of Physics: Conference Series **1752** (2021) 012005 doi:10.1088/1742-6596/1752/1/012005

Let $p^x_{(\alpha,\beta)}$ be an IFP of U, and $\hat{A}=(x,\mu_{\hat{A}},\gamma_{\hat{A}})$ be an IFS in U, we have $p^x_{(\alpha,\beta)}\in\hat{A}$ if $\alpha\le\mu_{\hat{A}}$ and $\beta\ge\gamma_{\hat{A}}$

Definition 3.4.
Let $\mathcal{B}$ be a collection of an IFS on U. Then $\mathcal{B}$ is said to be base for an IFT on U, if it satisfies the following:

1. $\forall p^x_{(\alpha,\beta)}\in U \ \ \exists\hat{B}\in\mathcal{B} \ \ni p^x_{(\alpha,\beta)}\in\hat{B}$
2. Let $\hat{B}_1,\hat{B}_2\in\mathcal{B}$ and $p^x_{(\alpha,\beta)}\in\hat{B}_1\cap\hat{B}_2$, then $\exists\hat{B}_3\in\mathcal{B} \ \ni p^x_{(\alpha,\beta)}\in\hat{B}_3\subseteq\hat{B}_1\cap\hat{B}_2$

Definition 3.3.
Let $\hat{A}=(x,\mu_{\hat{A}},\gamma_{\hat{A}})$ be an IFS in U and (U,τ) is an IFTS. We define intuitionistic fuzzy interior of $\hat{A}$ and intuitionistic fuzzy closure of $\hat{A}$, denoted by $i(\hat{A})$ and $c(\hat{A})$, receptively, as follows:
$i(\hat{A})=\bigcup\{O\mid O\in\tau\, and\, O\subseteq\hat{A}\}$ and $c(\hat{A})=\bigcap\{C\mid C\in\overline{\tau}\, and\, \hat{A}\subseteq C\}$

Proposition 3.4.
Let $\hat{A}=(x,\mu_{\hat{A}},\gamma_{\hat{A}})$ be an IFS in U. Then $c(\overline{\hat{A}})=\overline{\operatorname{int}(\hat{A})}$ and $i(\overline{\hat{A}})=\overline{c(\hat{A})}$

Proof.
Let the collection $\{(x,\mu_{\hat{O}_i},\gamma_{\hat{O}_i})\mid i\in J\}$ be the collection of IFOSs contained in $\hat{A}$. Then we have
$i(\hat{A})=\bigcup\{(x,\mu_{\hat{O}_i},\gamma_{\hat{O}_i})\}=\{(x,\max_i\mu_{\hat{O}_i},\min_i\gamma_{\hat{O}_i})\}$ and hence $\overline{i(\hat{A})}=\{(x,\min_i\gamma_{\hat{O}_i},\max_i\mu_{\hat{O}_i})\}$.
Because of $\overline{\hat{A}}=(x,\gamma_{\hat{A}},\mu_{\hat{A}})$ and $\mu_{\hat{O}_i}\le\mu_{\hat{A}},\ \gamma_{\hat{O}_i}\ge\gamma_{\hat{A}}\ \forall i\in J$ then $\{(x,\gamma_{\hat{O}_i},\mu_{\hat{O}_i})\}$ is the collection of IFCS containing $\overline{\hat{A}}$, i.e. $c(\overline{\hat{A}})=(x,\min_i\gamma_{\hat{O}_i},\max_i\mu_{\hat{O}_i})$. Hence $c(\overline{\hat{A}})=\overline{i(\hat{A})}$.
This is analogous to proof of $i(\overline{\hat{A}})=\overline{c(\hat{A})}$

Proposition 3.5.
Let $\hat{A}$ and $\hat{B}$ be IFSs in an IFTS (U,τ). We have the following properties:

1. $i(\hat{A})\subseteq\hat{A}$ and $\hat{A}\subseteq c(\hat{A})$
2. If $\hat{A}\subseteq\hat{B}$ then $i(\hat{A})\subseteq i(\hat{B})$ and If $\hat{A}\subseteq\hat{B}$ then $c(\hat{A})\subseteq c(\hat{B})$
3. $i(i(\hat{A}))\subseteq i(\hat{A})$ and $c(c(\hat{A}))\subseteq c(\hat{A})$
4. $i(\hat{A}\cap\hat{B})=i(\hat{A})\cap i(\hat{B})$ and $c(\hat{A}\cup\hat{B})=c(\hat{A})\cup c(\hat{B})$
5. $i(1_{\square})=1_{\square}$ and $c(0_{\square})=0_{\square}$

4. Intuitionistic fuzzy neighborhood and fuzzy continuity

Definition 4.1.
Let (U,τ) be an IFTS and $p^x_{(\alpha,\beta)}$ an IFP of (U,τ). Then an intuitionistic fuzzy neighborhood (IFN in short) of the IFP $p^x_{(\alpha,\beta)}$ is an IFS $\hat{A}$ such that $p^x_{(\alpha,\beta)}\in\hat{B}\subseteq\hat{A}$ with $\hat{B}\in\tau$.

Theorem 4.2.
Let (U,τ) be an IFTS and $\hat{A}$ be an IFS of U. Then we have
$\hat{A}$ is an τ-IFOS iff $\hat{A}$ is an IFN of $p^x_{(\alpha,\beta)}$, $\forall p^x_{(\alpha,\beta)}\in\hat{A}$

Proof.
$(\Rightarrow)$ Let $\hat{A}$ is an τ-IFOS, then $\hat{A}$ is an IFN $\forall p^x_{(\alpha,\beta)}\in\hat{A}$

Journal of Physics: Conference Series **1752** (2021) 012005 doi:10.1088/1742-6596/1752/1/012005

($\Leftarrow$) Suppose that $\hat{A}$ is an IFN $\forall p^x_{(\alpha,\beta)} \in \hat{A}$. Then $\exists \hat{B}_{p^x_{(\alpha,\beta)}}$ (τ-IFOS in U) $\ni$ $p^x_{(\alpha,\beta)} \in \hat{B} \subseteq \hat{A}$. So we have $\hat{A} = \bigcup\{p^x_{(\alpha,\beta)} \mid p^x_{(\alpha,\beta)} \in \hat{A}\} \subseteq \bigcup\{\hat{B}_{p^x_{(\alpha,\beta)}} \mid p^x_{(\alpha,\beta)} \in \hat{A}\} \subseteq \hat{A}$. Because each $\hat{B}_{p^x_{(\alpha,\beta)}}$ is an τ-IFOS, then $\hat{A}$ is an τ-IFOS in U too.

Definition 4.3.

Let (U,τ) and (V,λ) be two IFTSs, and given a function $f: U \to V$. We say that f is a fuzzy continuous function if and only if the preimage of each IFS in (V,λ) is an IFS in (U,τ).

The function f will be denoted as fuzzy open function if and only if the image of each IFS in τ is an IFS in λ.

Proposition 4.4.

Given a function $f:(U,\tau)\to(V,\lambda)$, then f is a fuzzy continuous iff the preimage of every IFCS in λ is an IFCS in τ.

Proof.

($\Rightarrow$) Suppose that $f:(U,\tau)\to(V,\lambda)$ is a fuzzy continuous, and given IFS $\hat{B} = (y,\mu_{\hat{B}},\gamma_{\hat{B}})$ in λ, and IFS $\bar{\hat{B}} = (y,\gamma_{\hat{B}},\mu_{\hat{B}})$ is the complement of $\hat{B}$ (so $\bar{\hat{B}}$ is an IFCS in λ). We have $f^{-1}(\bar{\hat{B}}) = (x, f^{-1}(\gamma_{\hat{B}}), f^{-1}(\mu_{\hat{B}})) = \overline{f^{-1}(\hat{B})}$ (proposisi…?), so by definition (4.3) $f^{-1}(\bar{\hat{B}}) = \overline{f^{-1}(\hat{B})} \in \tau$

($\Leftarrow$) Let $f:(U,\tau)\to(V,\lambda)$ be a function and the preimage of each IFCS in λ be an IFCS in τ. Consider $\hat{B} = (y,\mu_{\hat{B}},\gamma_{\hat{B}})$ is an IFS in λ, then the complement of $\hat{B}$, i.e. $\bar{\hat{B}} = (y,\gamma_{\hat{B}},\mu_{\hat{B}})$, is an λ-IFCS. $f^{-1}(\bar{\hat{B}}) = (x, f^{-1}(\gamma_{\hat{B}}), f^{-1}(\mu_{\hat{B}})) = \overline{f^{-1}(\hat{B})}$. Because $f:(U,\tau)\to(V,\lambda)$ is a function then $f^{-1}:(V,\lambda)\to(U,\tau)$ is also a function, so that $\hat{B} = (y,\mu_{\hat{B}},\gamma_{\hat{B}})$ is an IFCS in λ. So $f^{-1}(\hat{B}) = \overline{f^{-1}(\hat{B})}$ is an IFCS in U so that $f^{-1}(\hat{B}) \in \tau$. Hence f is the fuzzy continuous.

Proposition 4.5.

The following are equivalent each other.

1. $f:(U,\tau)\to(V,\lambda)$ is fuzzy continuous
2. $f^{-1}(i(\hat{B})) \subseteq i(f^{-1}(\hat{B}))$ $\forall \hat{B} \in V$
3. $c(f^{-1}(\hat{B})) \subseteq f^{-1}(c(\hat{B}))$ $\forall \hat{B} \in V$

5. Intuitionistic fuzzy compactness and fuzzy c₅-connectedness

Definition 5.1.

Let (U,τ) be an IFTS, we have

1. If $\hat{G}_i = \{(x,\mu_{\hat{G}_i},\gamma_{\hat{G}_i}) \mid i \in J\}$ is a collection of τ-IFOSs in U, then $\hat{G}_i$ is called fuzzy open cover of U if it satisfies the condition $\bigcup \hat{G}_i = 1$

A finite sub collection of a fuzzy open cover of U (it is also a fuzzy open cover of U) is called a finite sub cover.

2. A collection $\{(x,\mu_{\hat{K}_i},\gamma_{\hat{K}_i}) \mid i \in J\}$ of τ-IFCS in U satisfies the finite intersection property or FIP for short if and only if every finite sub collection $\{(x,\mu_{\hat{K}_i},\gamma_{\hat{K}_i}) \mid i = 1,2,\dots,n\}$ of the collection satisfies the condition $\bigcap_{i=1}^{n}\{(x,\mu_{\hat{K}_i},\gamma_{\hat{K}_i}) \mid i \in J\} \neq 0$

Definition 5.2.

Let (U,τ) be an IFTS, then it is called fuzzy compact if and only if every fuzzy open cover of U has a finite sub cover.

Proposition 5.3.

An IFTS (U,τ) is a fuzzy compact if and only if the IFTS $(U,\tau_{0.1})$ is fuzzy compact.
Proof.
$(\Rightarrow)$ Let (U,τ) be a fuzzy compact and let $\{[]\hat{G}_j \mid j \in K\}$ be a fuzzy open cover of U in $(U,\tau_{0.1})$.
Because of $\bigcup_j \{[]\hat{G}_j \mid j \in K\} = 1$, then we have $\max \mu_G = 1$. By $\gamma_{G_j} \le 1-\mu_{G_j}$ then $\min \gamma_{G_j} \le 1 - \max \mu_{G_j}$ = 1 – 1 = 0. So we have $\min \gamma_{G_j} = 0$. Hence $\bigcup_j \hat{G}_j = 1$. Because (U,τ) is fuzzy compact then $\exists \hat{G}_1, \hat{G}_2, ..., \hat{G}_{\bar{n}} \ni \bigcup_{i=1}^{n} \hat{G}_i = 1$, so we obtain $\max_{i=1}^{n}(\mu_{G_i}) = 1$ and $\min_{i=1}^{n}(1-\mu_{G_i}) = 0$. Hence, $(U,\tau_{0.1})$ is fuzzy compact.
$(\Leftarrow)$ Let $(U,\tau_{0.1})$ be fuzzy compact and let $\{\hat{G}_j \mid j \in K\}$ be a fuzzy open cover of U in (U,τ). Because of $\bigcup_{j=1} \hat{G}_j = 1$, then $\max \mu_{G_j} = 1$ and $\min(1-\mu_{G_j}) = 0$. Because $(U,\tau_{0.1})$ is fuzzy compact, then $\exists \hat{G}_1, \hat{G}_2, ..., \hat{G}_{\bar{n}} \ni \bigcup_{i=1}^{n} \{[]\hat{G}_i\} = 1$, so we obtain $\max_{i=1}^{n}(\mu_{G_i}) = 1$ and $\min_{i=1}^{n}(1-\mu_{G_i}) = 0$. By $\mu_{G_i} \le 1-\gamma_{G_i}$, then $1 = \max_{i=1}^{n} \mu_{G_i} \le 1 - \min_{i=1}^{n} \gamma_{G_i}$, so we have $\min_{i=1}^{n} \gamma_{G_i} = 0$. Hence $\bigcup_{i=1}^{n} \hat{G}_i = 1$, and therefore (U,τ) is fuzzy compact.
[22] introduced fuzzy C_5-connected concept, and [15] used the concept in IFS.
Definition 5.4.
Let (U,τ) be an IFTS. Then

1. U is called fuzzy C_5-disconnected if $\exists (\text{IFOS and IFCS}) \hat{G} \ni \hat{G} \ne 1$ and $\hat{G} \ne 0$
2. U is called fuzzy C_5-connected if U is not fuzzy C_5-disconnected.

Proposition 5.5.
Let (U,τ) be an IFTS, then U is fuzzy C_5-disconnected if and only if there exists a fuzzy continuous function $f:(U,\tau) \to (I_D,\tau_D)$ with $f \ne 0$ and $f \ne 1$
Corollary 5.6
Let (U,τ) be an IFTS, then U is fuzzy C_5-connected if and only if there does not exists fuzzy continuous function $f:(U,\tau) \to (I_D,\tau_D)$ with $f \ne 0$ and $f \ne 1$
Proposition 5.7.
Let (U,τ) and (V,λ) be two IFTSs, and given a fuzzy continuous surjection $f:U \to V$. If (U,τ) is fuzzy C_5-connected, then (V,λ) is fuzzy C_5-connected too.

References

[1] Zadeh L A 1965 Fuzzy sets *Information and Control* **8** 338-352
[2] Chang C L 1968 Fuzzy topological spaces *J. Math. Anal. Appl.* **24** 182-190
[3] Rosenfeld A 1971 Fuzzy Groups *J. Math. Anal Appl.* **35** 512-517
[4] Rosenfeld A 1975 Fuzzy Graphs in: L.A. Zadeh, K.S. Fu, M. Shimura (Eds.) *Fuzzy Sets and Their Applications* (New York: Academic Press) pp 77-95
[5] Olgun M, Ünver M and Yardımcı Ş 2019 Pythagorean fuzzy topological spaces *Complex Intell. Syst.* **5** 177
[6] Zhang D and Zhang G 2019 Continuous triangular norm based fuzzy topology *Arch. Math. Logic*
[7] Warren and Richard H 1983 Convergence in fuzzy topology *Rock. Mount. J. Math.* **13** 31-36.
[8] Krishna M and Marapureddy R 2018 Fuzzy graph of semigroup *Bull. Int. Math. Virt. Inst.* **8** 439-448

[9] Tom M and Sunitha M S 2015 Strong sum distance in fuzzy graphs *Springerplus* **4** 214
[10] Yuan X and Lee E S 2004 Fuzzy group based on fuzzy binary operation *Computers and Mathematics with Applications* **47** 631-641
[11] Akagul M 1988 Some properties of fuzzy groups *J. Math. Anal Appl.* **133** 93-100
[12] Atanassov K 1983 Intuitionistic fuzzy sets *VII ITKR's Session* (*Sofia*)
[13] Atanassov K 1986 Intuitionistic fuzzy sets *Fuzzy Sets and Systems* **20** 87-96
[14] Chaira T 2015 Thresholding of pathological images using Atanassov's intuitionistic fuzzy set *Int. J. Medic. Eng. Inf.* **7** 101-109
[15] Coker D 1997 An introduction to intuitionistic fuzzy topological spaces *Fuzzy Sets and Systems*. **88** 81-89
[16] Kim YC and Abbas S E 2005 Connectedness in intuitionistic fuzzy topological spaces *Commun. Korean Math. Soc.* **20** 117-134
[17] Lee S J and Lee E P 2000 The category of intuitionistic fuzzy topological spaces *Bull. Korean Math. Soc.* **37** 63-76
[18] Turanli N and Coker D 2000 Fuzzy connectedness in intuitionistic fuzzy topological spaces *Fuzzy Sets and Systems* **116** 369-375
[19] Park J H 2004 Intuitionistic fuzzy metric spaces *Chaos Solitons Fractals* **22** 1039–1046.
[20] Yan C, Wang X, and Nanjing, Intuitionistic i-fuzzy topological spaces *Czech. Math. J*, **60** 233–252
[21] Brown L M 1992 Fuzzy ditopological spaces *Proc. 2nd Int. Conf. of the Bulkanic Union for Fuzzy System and Artificial Intelligence* (*BUFSA*) (*Tranzon*) pp 142-145.
[22] Chaudhuri A K and Das P 1992 Fuzzy connected sets in fuzzy topological spaces *Fuzzy Sets and System* **49** 223 – 229

An Over view on intuitionistic fuzzy topological spaces To

ORIGINALITY REPORT

20%	19%	22%	14%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	Submitted to Far Eastern University Student Paper	3%
2	www.eudoxuspress.com Internet Source	3%
3	Emad A. Az-Zo'bi, Mohammad F. Marashdeh, Rama F. Uzbashy. "The fundamental group of intuitionistic fuzzy topological spaces", Applied Mathematical Sciences, 2014 Publication	2%
4	downloads.hindawi.com Internet Source	2%
5	lppm.ibrahimy.ac.id Internet Source	2%
6	Submitted to Higher Education Commission Pakistan Student Paper	2%
7	www.math.auckland.ac.nz Internet Source	2%
8	Submitted to Tikrit University Student Paper	2%

9	LİANG, Chengyu and YAN, Conghua. "Base and subbase in intuitionistic I-fuzzy topological spaces", Hacettepe Üniversitesi, 2014. Publication	2%

Exclude quotes	On	Exclude matches	< 2%
Exclude bibliography	On		