Performa Restricted Maximum Likelihood and Maximum Likelihood Estimators on Small Area Estimation

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Performa Restricted Maximum Likelihood and Maximum Likelihood Estimators on Small Area Estimation

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Abstract. In this paper, it was studied performance of maximum likelihood and restricted maximum likelihood method to estimate random factor on small area estimation. To evaluate of their performances, it was used mean square error as criteria. Based on data simulation, it was found that mean square error of restricted maximum likelihood estimator smaller than ML. Therefore, it could be concluded that restricted maximum likelihood method is better than maximum likelihood. Hereafter, it was applied those methods to human development index in South Sulawesi. The result of study showed that restricted maximum likelihood method is also better than Ordinary Least Square or maximum likelihood. The results of the study on human development index data in South Sulawesi Province; Gross Regional Domestic Product is one of the components that significantly affect the human development index.

1. Introduction

Small area estimation (SAE) is a topic of great importance due to the growing demand for reliable small area statistics even when only very small samples are available for these areas [1]. Small area estimation (SAE) is a statistical technique to estimate parameters of sub-population whose small sample size. The terms small area and local area are commonly used to denote a small geographical area, such as a county, a municipality or a census division [2]. Small area meant as a subset of population whose small sample size with a variable that paid attention to [2]. The election of estimation method on the random factor of right small area model is able to increase the prediction accuracy of parameter area that is concern.

The accuracy of prediction result via mean square error (MSE) of SAE had been supported by some previous researchers [3]. Datta and Lahiri studied on the properties of MSE unfamiliar second-order of best linear unbiased prediction (BLUP) estimator [4]. The related other studies of MSE on SAE [5,6,7].

They said that BLUP estimator minimizes MSE, such as unbiased estimator group does not depend on normality of random affect factor, but relies on random factor variance that estimated through method of fitting constants or moment. Other alternatives in getting of estimation variance and covariance from random factor are to utilize the methods of maximum likelihood (ML) and Restricted Maximum Likelihood (REML), assuming normality. REML estimation is a method employed to estimate variance-covariance parameters from data that follow a Gaussian linear model [8].

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These two methods do not look their difference of accuracy result when big enough-taken sample size. Those occurred because both of them would be asymptotically convergent for the amount of definite auxiliary variables.

This asymptotical convergent becomes an interesting thing because of the model built up based on the small area. Accordingly, it needs a specific study to know how well the two methods of random factor estimation, ML and REML are, related to various domains taken.

2. Preliminary

This small area is described as a small geographic area or subpopulation from a demographic group. Moreover, small area as a description of a small geographic area could be exemplified like province or district in a country. While, subpopulation from a demographic group, the small area could be race, gender, and age in a wider geographic area [9].

SAE as a statistical technique could be applied in two ways, direct and indirect estimation. In the case of area whose a small sample size, the direct estimation tends to produce an error, to overcome it, that is done the indirect estimation. The small area model-based indirect estimation is called model-based estimator.

2.1. Area-Based Model

Area-based model or Fay-Herriot model is based on the availability of supporting data only on the area level. Such as $\mathbf{x}_i = (x_{1i}, \cdots, x_{pi})^T$ for $i = 1, 2, \cdots, m$ as supporting variable on area to-i, then area-based model said as

$$\boldsymbol{\theta}_i = \mathbf{X}_i^T\boldsymbol{\beta} + \boldsymbol{z_i}\boldsymbol{\nu_i} \quad (1)$$

with $\boldsymbol{\theta}_i$ is variable of area response to-i , $\mathbf{X}_i$ is the vector of area supporting variable to-i that has the measurement of px1, $\boldsymbol{\beta}$ is parameter vector of supporting variable that sized px1, $\boldsymbol{z_i}$ is constant of positive random effect on area to-i and $\boldsymbol{\nu_i}$ is the *random effect* on area to-i. for instance, $\boldsymbol{\theta}_i$ is the estimator for $\boldsymbol{\theta}_i$, $\boldsymbol{\theta}_i$could be said as

$$\boldsymbol{\theta}_i = \boldsymbol{\theta}_i + \boldsymbol{e}_i \quad (2)$$

with $\boldsymbol{e}_i$ as *sampling error area to-i, i*=1,2,...,m. Substitute for the model (1) to the model (2) will result in

$$\boldsymbol{\theta}_i = \mathbf{X}_i^T\boldsymbol{\beta} + \boldsymbol{z_i}\boldsymbol{\nu_i} + \boldsymbol{e}_i \quad (3)$$

with $\boldsymbol{e}_i$ assumed to have the normal distribution with mean 0 and variance of σ_i^2, and $cov(\boldsymbol{\nu_i}, \boldsymbol{e}_i) = 0$.

2.2. Empirical Best Linear Unbiased Prediction Estimator

Notice to the model (3), and parameter linear combination of definite influence of $\boldsymbol{\beta}$ and parameter of random influence of $\boldsymbol{\nu_i}$, $\boldsymbol{\theta}_i = \mathbf{X}^T\boldsymbol{\beta} + \boldsymbol{z_i}\boldsymbol{\nu_i}$. Best linear unbiased prediction (BLUP) estimator for $\boldsymbol{\theta}_i$ is $\theta_i = x_i^T\beta + z_i^T\nu$, with $\nu_i = \frac{\tau^2 Z_i}{\sigma_i^2 + Z_i^2\tau^2}(\theta_i - \mathbf{x}_i\boldsymbol{\beta})$,

$\boldsymbol{\beta} = \left(\sum_{i=1}^{m} \frac{\mathbf{x}_i\mathbf{x}_i^T}{\sigma_i^2 + \tau^2 Z_i^2}\right)^{-1} \sum_{i=1}^{m} \frac{\mathbf{x}_i\theta_i}{\sigma_i^2 + \tau^2 Z_i^2}$

or could be stated as

$$\theta_i^{BLUP} = \gamma_i\boldsymbol{\theta}_i + (1 - \gamma_i)\mathbf{X}_i^T\boldsymbol{\beta} \quad (4)$$

with $\gamma_i = \frac{Z_i^2\tau^2}{\sigma_i^2 + \tau^2 Z_i^2}$. If τ^2 is not known, then it should be estimated. For instance, τ^2 is unbiased estimator for τ^2, and $\gamma_i = \frac{Z_i^2\tau^2}{C + \tau^2 Z_i^2}$, then empirical best linear unbiased prediction (EBLUP) for $\boldsymbol{\theta}_i$,

$$\theta_i^{EBLUP} = \gamma_i\boldsymbol{\theta}_i + (1 - \gamma_i)\mathbf{X}_i^T\boldsymbol{\beta} \quad (5)$$

with dengan $\boldsymbol{\beta} = \left(\sum_{i=1}^{m} \frac{\mathbf{x}_i\mathbf{x}_i^T}{\sigma_i^2 + \tau^2 Z_i^2}\right)^{-1} \sum_{i=1}^{m} \frac{\mathbf{x}_i\theta_i}{\sigma_i^2 + \tau^2 Z_i^2}$

2.3. Variant Estimation of Random Influence

Factor of the random influence on the area small model is represented by variance of every area. In model (3), assumed that there is the local variation, $\tau^2 > 0$ to estimate τ^2. Rao proposed on scoring algorithm in estimating the parameter τ^2, In which this relied on the ML and the REML [9]. S*coring algorithm* of ML defined as

$$\tau^2_{a+1} = \tau^2_a + I\left(\tau^2_a\right)^{-1} S\left(\tau^2_a\right) \quad (6)$$

where $I(\tau^2) = \frac{1}{2}\sum_{i=1}^{m} \frac{z_i^4}{(z_i^2\tau^2+\sigma_i^2)^2}$ and

$$S(\tau^2) = -\frac{1}{2}\sum_{i=1}^{m} \frac{z_i^2}{z_i^2\tau^2+\sigma_i^2} + \frac{1}{2}\sum_{i=1}^{m} \frac{z_i^2(\theta_i - x_i^T\beta)^2}{(z_i^2\tau^2+\sigma_i^2)^2}$$

while scoring *algorithm* of REML defined as

$$\tau^2_{a+1} = \tau^2_a + I_R\left(\tau^2_a\right)^{-1} S_R\left(\tau^2\right) \quad (7)$$

where τ^2_{a+1} is estimator of a[th] iteration of τ^2,

$I_R(\tau^2) = \frac{1}{2}tr(PBPB)$,

$S_R(\tau^2) = -\frac{1}{2}tr(PB) + \frac{1}{2}\theta^T PBP\theta$, $B = diag(z_i^2)$ and

$P = V^{-1} - V^{-1}X(XV^{-1}X^T)^{-1}X^TV^{-1}$

Methods of ML and REML based on model (7) and model (8) have the similarity. It is necessary to further investigate in order to recognize the relevant data characteristic for both methods. In order to size the benefit of method in model, it will be tested to occupy the criteria of mean square error from EBLUP of ML and REML related to simulation of data. MSE of EBLUP of a prediction on the area model illustrated as

$$MSE\left(\theta_i^{EBLUP}\right) = g_{1i}(\tau^2) + g_{2i}(\tau^2) + 2g_{3i}(\tau^2) \quad (8)$$

where $g_{1i}(\tau^2) = \frac{z_i^2\tau^2\sigma_i^2}{\sigma_i^2+z_i^2\tau^2}$

$$g_{2i}(\tau^2) = \left(\frac{\sigma_i^2}{\tau^2 z_i^2+\sigma_i^2}\right)^2 \mathbf{X}_i^T \left(\sum_{i=1}^{m} \frac{\mathbf{X}_i\mathbf{X}_i^T}{\sigma_i^2+z_i^2\tau^2}\right)^{-1} \mathbf{X}_i$$

and $g_{3i}(\tau^2) = \frac{\sigma_i^2 z_i^2}{(\tau^2 z_i^2+\sigma_i^2)^3} Var(\tau^2)$

$\sigma_i^2 = Var(e_i)$, $\tau^2 = Var(v_i)$And $z_i$is constant that corresponding with random factor of v_i.

3. Main Results

3.1. Estimations of ν and β

Pay attention again to area small model (3), $\boldsymbol{\theta}_i = \mathbf{X}^T\boldsymbol{\beta} + \mathbf{z}_i\boldsymbol{\nu}_i + \mathbf{e}_i$, i=1,2,...m. or in the vector form, it could be described as

$$\boldsymbol{\theta} = X\boldsymbol{\beta} + Z\nu + e \quad (9)$$

with $\nu \sim N(0, D)$, $D = I\tau^2$, $e \sim N(0, G)$, $G = I\sigma_i^2$

$E(\theta) = X\boldsymbol{\beta}$ and $Var(\theta) = V = D + ZGZ^T$ in connected to model (9), it was obtained the function of likelihood

$$L(\boldsymbol{\beta}, \boldsymbol{\delta}) = (2\pi)^{-\frac{m}{2}} |V|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\theta - X\boldsymbol{\beta})^T V^{-1}(\theta - X\boldsymbol{\beta})\right)$$

and, where $\boldsymbol{\delta} = \tau^2, \sigma^2{}_i$ To achieve estimator for$\boldsymbol{\beta}$, differentiation was done $V^{-1}X$. Such as $\frac{\partial \ln L(\boldsymbol{\beta},\boldsymbol{\delta})}{\partial \boldsymbol{\beta}} = \theta^T V^{-1}X - (X\boldsymbol{\beta})^T\boldsymbol{\beta}$ was the estimator of $\boldsymbol{\beta}$ to maximize $\ln L(\boldsymbol{\beta}, \boldsymbol{\delta})$, so that it was gotten

$$\theta^T V^{-1}X - X\boldsymbol{\beta}^T V^{-1}X = 0$$

$\boldsymbol{\beta} = X^T V^{-1} X^{-1} X^T V^{-1} \boldsymbol{y}$ ■

To get the estimator for $\boldsymbol{\nu}$, Let E $\theta \ \boldsymbol{\nu} = X\boldsymbol{\beta} + Z\boldsymbol{\nu}$ and $Var\ \theta \ \boldsymbol{\nu} = G$. Connected to above, it was gained

$$L\ \boldsymbol{\beta}, \boldsymbol{\delta}\ \boldsymbol{\nu} = 2\pi^{-\frac{m}{2}} G^{-\frac{1}{2}} \exp -\frac{1}{2}\ \theta - X\boldsymbol{\beta} - Z\boldsymbol{\nu}\ ^T G^{-1}\ \theta - X\boldsymbol{\beta} - Z\boldsymbol{\nu} \quad (10)$$

$$L\ \boldsymbol{\nu} = 2\pi^{-\frac{m}{2}} D^{-\frac{1}{2}} \exp -\frac{1}{2}\boldsymbol{\nu}^T D^{-1}\boldsymbol{\nu} \quad (11)$$

From (10) and (11), it was found

$$L\ \boldsymbol{\beta}, \boldsymbol{\delta}, \boldsymbol{\nu} = 2\pi^{-\frac{m}{2}} G^{-\frac{1}{2}} \exp -\frac{1}{2}\ \theta - X\boldsymbol{\beta} - Z\boldsymbol{\nu}\ ^T G^{-1}\ \theta - X\boldsymbol{\beta} - Z\boldsymbol{\nu}\ 2\pi^{-\frac{m}{2}} D^{-\frac{1}{2}} \exp -\frac{1}{2}\boldsymbol{\nu}^T D^{-1}\boldsymbol{\nu}$$

and $\ln L\ \boldsymbol{\beta}, \boldsymbol{\delta}, \boldsymbol{\nu} = -m \ln 2\pi - \frac{1}{2} G - \frac{1}{2}\ \theta - X\boldsymbol{\beta} - Z\boldsymbol{\nu}\ ^T G^{-1}\ \theta - X\boldsymbol{\beta} - Z\boldsymbol{\nu} - \frac{1}{2}\ln D - \frac{1}{2}\boldsymbol{\nu}^T D^{-1}\boldsymbol{\nu}$

To acquire the estimator for $\boldsymbol{\nu}$, the following is the function of $\ln L\ \boldsymbol{\beta}, \boldsymbol{\delta}, \boldsymbol{\nu}$ differentiated toward $\boldsymbol{\nu}$,

$$\frac{\partial \ln L\ \boldsymbol{\beta}, \boldsymbol{\delta}, \boldsymbol{\nu}}{\partial \boldsymbol{\nu}} = -\frac{1}{2}\frac{\partial}{\partial \boldsymbol{\nu}}\ \theta - X\boldsymbol{\beta} - Z\boldsymbol{\nu}\ ^T G^{-1}\ \theta - X\boldsymbol{\beta} - Z\boldsymbol{\nu} - \frac{1}{2}\frac{\partial}{\partial \boldsymbol{\nu}}\boldsymbol{\nu}^T D^{-1}\boldsymbol{\nu}$$
$$= \boldsymbol{\theta} - \boldsymbol{X\beta} - \boldsymbol{Z\nu}\ \boldsymbol{G}^{-1}\ \boldsymbol{Z}^T\boldsymbol{\nu}\ \boldsymbol{D}^{-1}. \quad (12)$$

Let $\boldsymbol{\nu}$ is the estimator for $\boldsymbol{\nu}$ that maximize the function of $\ln L\ \boldsymbol{\beta}, \boldsymbol{\delta}, \boldsymbol{\nu}$, so that

$$\frac{\partial \ln L\ \boldsymbol{\beta},\boldsymbol{\delta},\boldsymbol{\nu}}{\partial \boldsymbol{\nu}}_{\boldsymbol{\nu}=\boldsymbol{\nu}} \Leftrightarrow \ \theta - X\boldsymbol{\beta} - Z\boldsymbol{\nu}\ ^T G^{-1}\ \boldsymbol{Z} - \boldsymbol{\nu}\ ^T D^{-1}\ = 0,$$

$\boldsymbol{\nu} = DZ^T V^{-1}\ \theta - X\boldsymbol{\beta}$, where $V = D + ZGZ^T$.

3.2. Simulation

The data, in this simulation, were risen up to specify the mean square error of EBLUP of ML and REML with the variance of random effect of domains varied. The generated data followed the model of $\boldsymbol{\theta}_i = \mathbf{X}^T\boldsymbol{\beta} + z_i v_i + e_i$, $i=1,2,...,m$, by taking only one variable of X, and the intercept of 100, and be taken $z_i = 1$. Stages of simulation are as follows

1. Determine m = 20, 50, 150, 200 and 500
2. State β 2, generate $X_i \sim N\ 10,4$, $v_i \sim N\ 0, \tau^2$, $\tau^2 = 1,2$ $e_i \sim N\ 0,1$, i=1,2,...,m
3. Decide value of $\boldsymbol{\theta}_i$ through the equation of $\boldsymbol{\theta}_i = 100 + 2X_i + v_i + e_i$
4. Get sample fraction of 5%, 10%, 15% and 20% from the generated data on the third stage (3)
5. Use the formulation of model (8) to gain $MSE\ \theta_i^{EBLUP}$, for the estimator of τ^2 based on the methods of ML of equation (6) and REML of equation (7)
6. next, calculate the mean of MSE from m domain with the use of formulation $MSE\ \theta^{EBLUP} = \frac{1}{m} MSE\ \theta_i^{EBLUP}$
7. Do stage 1 until stage 2 until stage 6 in order to get B $MSE\ \theta^{EBLUP}$ and its average count

Here, we use MSE for measure accurate of SAE model because This measure to be satisfied by not only the maximum likelihood (ML) and restricted maximum likelihood (REML), but also other estimators including the Prasad-Rao and Fay-Herriot estimators in specific models [10].

3.3. Scenario I, $\tau^2 = 1$

On simulation 1, the random effect variance taken $\tau^2 = 1$. In related to the simulation 1's result data with 100 times repetition, gained $MSE\ \theta^{EBLUP}$. Table 1 presented MSE from the methods of ML and REML on EBLUP for every domain measure and sample fraction that tried.

In Table 1 below, it was found that the more increased the domain measure was, the smaller the MSE value for several sample fraction that applied. Furthermore, it could show that the larger the sample fractions were, the smaller the MSE for every certain domain measure that utilized. It could

also be paid attention based on Table 1 that for the all domain measure and sample fraction tried; the estimation REML method could give the smaller MSE relatively than the MSE from the ML method. From these proofs, it could be decided that for the model case with $\tau^2 = 1$, REML was the better choice.

Table 1. MSE for various domain measure and sample fraction

		fraction sample			
		5%	**10%**	**15%**	**20%**
m=20	ML	0,1658900	0,0917131	0,0635364	0,0478599
	REML	0,1650651	0,0913664	0,0633726	0,0477660
m=50	ML	0,1604874	0,0901828	0,0626057	0,0478015
	REML	0,1603439	0,0901344	0,0625829	0,0477886
m=100	ML	0,1584601	0,0894987	0,0621206	0,0477182
	REML	0,1584182	0,0894871	0,0621151	0,0477150
m=150	ML	0,1579101	0,0899223	0,0624929	0,0474889
	REML	0,1578884	0,0899170	0,0624904	0,0474875
m=200	ML	0,1574648	0,0895491	0,0618369	0,0474917
	REML	0,1574515	0,0895460	0,0618356	0,0474909
m=500	ML	0,1568777	0,0891648	0,0619705	0,0473549
	REML	0,1568740	0,0891642	0,0619703	0,0473548

3.4. . Scenario II, $\tau^2 = 2$

Table 2. MSE for various domain measure and sample fraction

		fraction sample			
		5%	**10%**	**15%**	**20%**
m=20	ML	0,1855630	0,0952171	0,0649718	0,0496187
	REML	0,1849459	0,0950274	0,0648843	0,0495639
m=50	ML	0,1780245	0,0949477	0,0647277	0,0486127
	REML	0,1779263	0,0949210	0,0647159	0,0486061
m=100	ML	0,1769534	0,0939382	0,0648016	0,0487011
	REML	0,1769288	0,0939317	0,0647987	0,0486995
m=150	ML	0,1761152	0,0942190	0,0643477	0,0489452
	REML	0,1761037	0,0942161	0,0643464	0,0489445
m=200	ML	0,1753076	0,0945942	0,0640756	0,0487721
	REML	0,1753010	0,0945925	0,0640749	0,0487717
m=500	ML	0,1751355	0,0946012	0,0642600	0,0486819
	REML	0,1751342	0,0946009	0,0642599	0,0486818

Referred to Table 2 above, it showed the same tendency of such Table 1 in the case $\tau^2 = 1$, in which the more increased the domain measure was, the smaller the MSE value for every certain domain measure that applied. The same things also occurred for the all domain measures and the sample fractions tried, the REML estimation method could supply the smaller MSE relatively than the MSE from the ML method. Connected to these statements, it could be summed that for the model case of$\tau^2 = 2$, REML was the better option.

3.5. Comparison Analysis of scenarios 1 and 2

The analysis study of scenarios 1 and 2 was focused on the tendency of RMSE for the two methods that resulted from the change of random effect. Graph 1 pointed the correlation between RMSE and sample fraction, estimation method and scale of random effect.

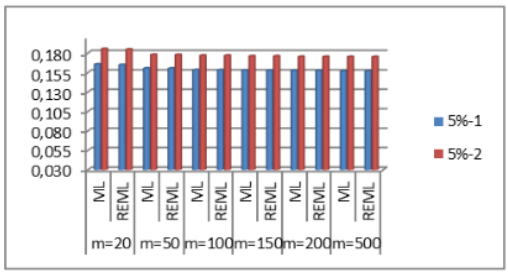

Figure 1a. Graph of 5% sample fraction

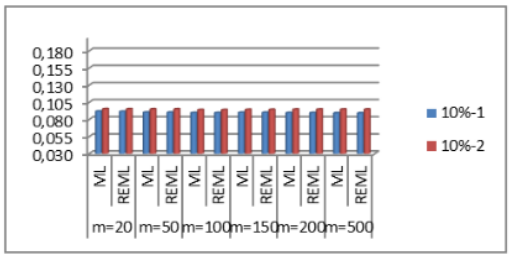

Figure 1b. Graph of 10% sample fraction

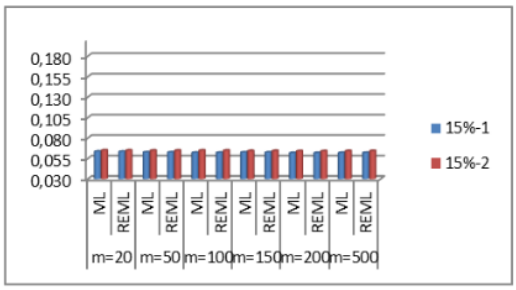

Figure 1c. Graph of 15% sample fraction

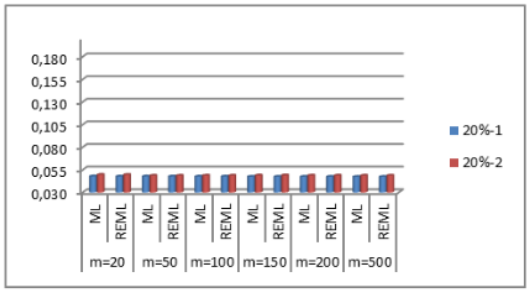

Figure 1d. Graph of 20% sample fraction

From the above first figure, it could be seen that the existence of the sample fraction improvement decreased the RMSE value for both of the ML and REML estimation methods. Besides, the scenario 1's RMSE value was always higher than scenario 2 from the second result of estimation method. However, the difference of estimation results among the two methods would gravitate same when the sample fraction improved. Referred to the simulation results on the scenarios 1 and 2 by using the various sample fractions, it could be specified that the REML result could always present the better results than the ML.

3.6. Comparison Analysis of South Sulawesi HDI Data

In Table 3 below described that the HDI estimation result with the REML method could offer the higher accuracy than the other two methods.

Table 3. Comparison of HDI Estimation Result on the methods of ML, OLS and REML

City Code	HDI actual	HDI-REML	HDI-ML	HDI-OLS
1	70.100	70.232	70.276	70.846
2	70.860	70.857	70.856	70.767
3	70.170	70.406	70.487	71.877
4	71.190	71.169	71.160	70.871
5	74.550	74.118	73.961	70.953
6	70.670	70.674	70.675	70.796
7	64.920	65.758	66.046	70.577
8	69.340	69.368	69.379	70.601
9	73.980	73.701	73.598	71.431
10	72.790	73.358	73.551	76.237
11	74.320	73.725	73.534	71.261
12	78.790	78.229	78.042	75.463

Moreover, it would be presented the ANOVA model result of SAE model for REML method in Table 4 below.

Table 4. ANOVA IPM metode REML

	beta	**Std.error**	**t-value**	***p*-value**
(Intercept)	68.2932	2.2763	30.0013	9.4433e-198
#poor	0.0118	0.0340	0.3468	7.2874e-010
grdp	0.2256	0.1096	2.0581	3.9585e-020

Concerning to Table 4 above, it could be figured out that from the variables of the poor and of gross regional domestic product (GRDP), the amounts of the poor were not significant. Intercepts were significant so much, in which it meant that it was possible that the potentials of each city or district

had the special characteristic, and also might still exist some independent variables enable included in the model to measure HDI.

4. Conclusions

Based on this analysis, we can conclude that: (1) small area estimator with the REML method offered more valid estimator than with the methods of ML and OLS; and (2) GDRB constituted the significantly influenced factor towards the improvement of HDI in South Sulawesi.

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