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"Science Enhancement for Developing Countries"

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HASANUDDIN UNIVERSITY

Proceeding

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PREFACE

Makassar city, the capital of South Sulawesi province known as one of the biggest cities in Indonesia and also having Hasanuddin University, the biggest university in eastern part of Indonesia, has plenty of natural resources and human resources. Having a strategic position at the center point of Indonesia, Makassar has been developing very rapidly, and has been contributing to the regional, national and even international economic development. Given this, science can play important roles and therefore is needed to support rapid development in various sectors.

With regard to this, cooperates with Ministry of Environment Indonesia, Atmospheric and Ocean Research Institute (AORI) Japan, University of Kebangsaan Malaysia (UKM), Alfred Wagener Institute (AWI) Germany, Queensland University of Technology (QUT) and Flinders University Australia, Faculty of Mathematics and Natural Sciences Hasanuddin University carried out "The First International Conference on Science (ICOS-1)" on November 19-20, 2014, in Hotel Clarion Makassar. The theme of ICOS-1 is "Science Enhancement for Developing Countries". The conference attended by two hundred participants and came from Asia (Japan, Malaysia, Indonesia), Australia, and Europe.

There are approximately 97 research articles for oral presentations and 16 poster presentations, ranging from Biology, Statistics, Mathematics, Chemistry, Physics, Geophysics, Computer Science and Environmental Science. Of the 113 papers, there are approximately 79 papers were selected to be published in the proceedings of the ICOS-1 through the peer review process.

With regard to the delivery of the ICOS-1 in 2014 and the completion of the proceedings ICOS-1, 2014, allow us to thanks to: the authors for providing the content of the program, the conference participants who came from several public and private universities, the program committee and the senior program committee, who worked very hard in reviewing papers and providing feedback for authors to be included in the Proceedings of ICOS-1, 2014, the hosting organisation Hasanuddin University, our keynote and invited talk presentations including Ir. Muh Ilham Malik M.Sc, from Ministry of Environment Indonesia, Prof Koji Inoue from AORI Japan, Prof Mohammad B Kassim from UKM Malaysia, Dr.rer.nat Dominik Kneer from AWI Germany, Prof Dadang A. Suriamihardja and Prof Alfian Noor from Hasanuddin University, Prof Kerrie Mengersen from QUT and Dr. Darfiana Nur from Flinders University, Australia.

Hopefully is of benefit to all readers.

Yours faithfuly,
Prof Dr. Hanapi Usman M.S
Dean of Faculty Mathematics and Natural Sciences
Hasanuddin University

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STUDENTS' DIFFICULTIES IN MATHEMATICS PROOFS

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Abstract

This qualitative research aimed at investigating difficulties experienced by students in Mathematics Department, State University of Makassar. This study was conducted in Mathematics Education Study Program at Mathematics Department, Faculty of Mathematics and Science, State University of Makassar in academic year 2013/2014. The subjects involved in the research were students of Mathematics Education Study Program in the subjects of Calculus, Trigonometry, and School Mathematics. The subjects were administered in tests concerning mathematics proof. Further, the students' answers were then categorized into several aspects of difficulties in proving. The research findings showed that there were four categories of difficulties experienced by the students in proving statements in mathematics. The four categories were:(1) the use of symbols which was inappropriate; (2) the lack of students' understanding about mathematics proof; (3) the difficulties in selecting strategies for mathematics proof; and (4) the lack of understanding of concepts and principles in mathematics.

Key Words: *Mathematics proof, difficulties, students.*

1. INTRODUCTION

In mathematical courses at universities, students strive to solve problems requiring them to prove, verify, justify, or show. Essentially, all these instructions ask them to prove mathematical statements. Proof is the most important tool in mathematics [6]. Proving is one of the main activities when someone is learning mathematics. However, difficulties in proving mathematical statements are encountered by most of the students. In the last few years of our teaching, we found that students showed symptoms of problems in proving mathematical statements. It is evident that some of them just take one or two examples or cases and verifying the truth of the statements on the cases carelessly lead them to conclude that the statements are true or already proven. It seems that they do not understand what constitutes a mathematics proof [16]. Besides, a conditional sentence confuse students and they cannot figure out where to start its proof and where it ends. Above all, they do not comprehend what attributes are assigned to a valid mathematics proof. In this research, the problem under investigation is what and how are the difficulties experienced by the students in proving mathematical statements. The findings will be beneficial for lecturers to further identify appropriate strategies to help the students succeed in proving mathematical statements.

2. PROVING AND PROOF IN LEARNING MATHEMATICS

2.1 Proving and Proof

Everyone learning mathematics, especially in higher level, must know what constitutes a proof, why proof is needed, and how to construct proof. All these are to lead the learners to comprehend the structure of mathematics. Proving is a challenging activity in learning (doing) mathematics. Constructing proof is not considered as a branch of mathematical activities. It is actually the essence of mathematics, and someone cannot be considered as learning mathematics unless they learn the 'what' and the 'how' of mathematics proof [2]. With a very close relationship to reasoning, both proof and reasoning are not merely an occasional activity done within a special time or a special topic in mathematics, but they should be a natural, ongoing component of learning activities [12]. Within the community of mathematics, there is no strict consensus about the meaning of proof, its role, and the way it is constructed, verified, and accepted [8].

Although students have been exposed to proofs in their school mathematics, they first meet the formal proof concept as accepted in the community of mathematicians at university level. Much of the proving process is a sequence of mental and physical actions, such writing or thinking of a statement in a proof, drawing or visualizing an illustration on the results of previous actions, or trying to remember an example [15]. It is useful to consider a whole range of roles proofs play in mathematical practices [6]. Some of them are verification, explanation, systematization, discovery/invention, communication, exploration, construction, and incorporation (see [13]). Learning and teaching are expected to reflect all the roles. It is argued that while in mathematical practice the main function of proof is verification and justification, its main role in mathematics education actually is explanation [5]. Accordingly, the need for understanding and the need for validity seem to be recognized as being the characteristics of the theoretical discussion about the nature and the function of proofs [10]. Some of the roles seem to be indistinguishable and overlap to one another [9]. Also, some of them exist in a close relationship to other functions such as: incorporation and systematization, or discovery and exploration.

2.2 Difficulties of Proving

In their school mathematics experiences, in general students struggle mostly with the computational aspect of mathematics and see mathematics as a list of facts, rules, and procedures. Therefore when they move to a university, they have difficulties in understanding the abstract and axiomatic structure of mathematics based on concepts, relations between concepts, definitions, theorems, and proofs. There have been seven major sources of student difficulties in proving [11], that is, inability to state the definition; lack of intuitive concept understanding; inadequate concept images of proving; inability to generate examples; inability to understand and use mathematical language and symbols; inability to utilize definitions to structure proofs; and inability to start proving.

The occurrence of student difficulties in proving are associated with factors, such as, the understanding of the rules and nature of proof; conceptual understanding supporting the proof; proving techniques and strategies; and cognitive load [4]. Reasoning errors and misconceptions also contribute to the difficulties in proving [14] including misconception in

categorizing definitions and misunderstanding about the mathematical language [3]. Further, the difficulties are caused by the imprecision in writing mathematics and lack of understanding about how to construct proofs by appropriately applying rules of logic [1]. There are two general issues concerning the students' difficulties in doing proof [10]. These issues correspond to two main questions, that is, what are the kinds of difficulties and which might be the origin of such difficulties. This research only covers the former question.

3. METHODOLOGY

This research is an explorative qualitative study aiming at exploring and describing the difficulties experienced by students in mathematics proof. It was conducted at Mathematics Education Study Program, Mathematics Department, State University of Makassar, Indonesia during the second semester of academic year 2013/2014. The research subjects were 121 students enrolled in three different units, that is, Calculus 2 (37 students), Trigonometry (31 students), and School Mathematics 1 (53 students). The instruments used in this study were written essay tests. The tests were administered both at the mid and the end of the semester. The students sit in the mid semester test in two units, that is, Calculus 2 and Trigonometry, and they sit in the final semester test in three units, that is, Calculus 2, Trigonometry, and School Mathematics 1. The students' answer sheets were then examined and an inductive analysis was employed focusing on the difficulties encountered by them in solving problems of mathematical proving. The difficulties were grouped based upon categories provided in the theoretical review. Besides, additional categories were made in order to cover the kinds of difficulties left uncovered in the initial categories. Further, the students' difficulties were described in the form of simple profiles accompanied by illustrations exemplifying students' difficulties which were quoted from mistakes performed in their answer sheets.

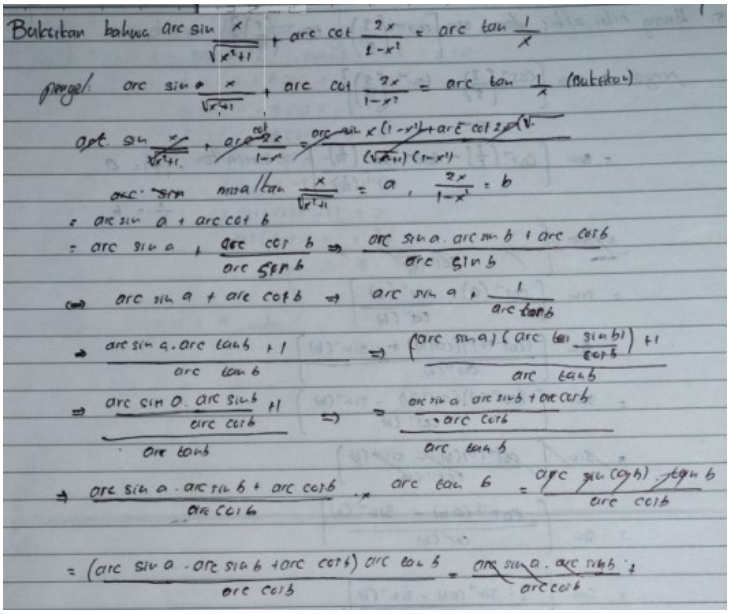
4. RESULTS AND DISCUSSION

4.1. Results

Based upon the inductive analysis, it was found that there were four categories of students' difficulties in mathematical proof.

4.1.1. Mathematical Symbols

Some students could not understand the meaning of mathematical symbols. Further, they did not use them appropriately. This caused the proof they constructed to be meaningless. The meaning of symbols in mathematics is very specific and unique, so their use

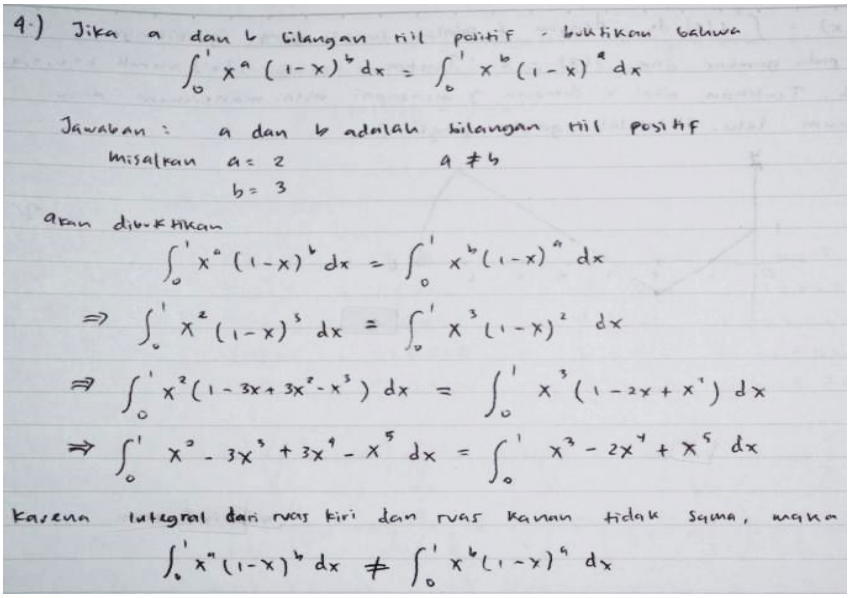


Student's Inappropriate Use of Mathematical Symbols

will very much influence the meaning of sentences. The use of implication and bi-implication in the proof as illustrated in the picture ruins the meaning of the sentence. The relationship between the components of the sentence, indeed, is equal (=), as they talks about real values; they are not equivalent sentences with their truth values. It was also found that the students were confused by the power of -1 where they considered $\cot^{-1}x$ as the reciprocal of cot x, that is, tan x.

4.1.2. Comprehension of Mathematics Proofs

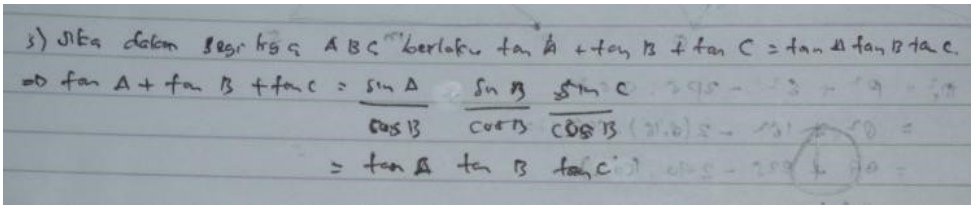
Students' seemed to experience difficulties in understanding what a mathematics proof is. Although a heuristic proof assisted by computer software could be accepted nowadays, logic involving a deductive axiomatic system still dominates the discussion about proving in mathematics. Some students still considered it enough just to give or show one example or case satisfying the statement to prove. In proving a trigonometry identity, some students tried to substitute certain real value into variable x. An understanding that verifying a case constitutes a poof sometimes leads students to a very clear correct statement. This fact turns to ensure him that the expected proof has been completed, while he forgets that the statement to prove applies for infinite number of cases. Students' understanding of proving an equity in mathematics was still problematic because they did not realize that proving an equity was about proving the value, not the form or appearance. The integrands can be different in appearance but the final results are equal to each other.



Student's Problem of What Constitutes a Mathematical Proof

The students also found it difficult to understand what was expected by the sentence to prove. They failed to comprehend a conditional sentence, its essence and how to prove it. It seems difficult for them to utilize the sufficient condition in the statement and to proceed through several steps to achieve the necessary condition or the conclusion of the statement.

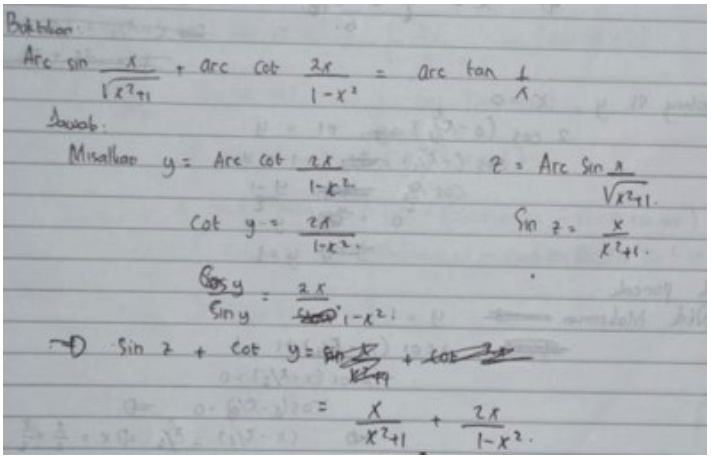
In illustration above, the student was just busy with the laws of tangent without capitalizing on the hypothesis that the angles under investigation were of a triangle.



Student's Misunderstanding of the Statement to be Proved

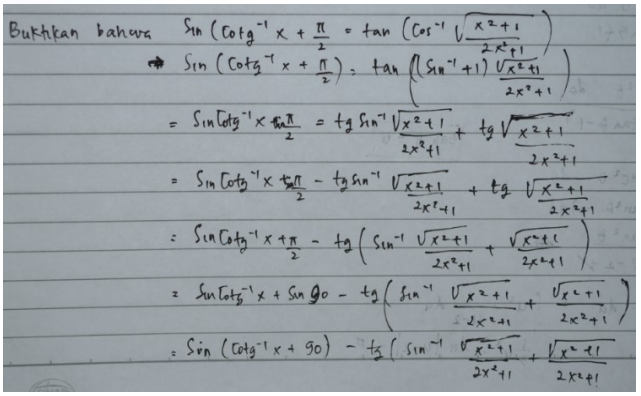
4.1.3. Strategies for Mathematical Proving

Students experienced problems in constructing the proof of a statement. They found it difficult to determine which strategy was appropriate to employ. For example, when they came to prove an equality. In the illustration, the first step was actually on the right track, namely, simplifying the problem by symbolizing, which should help the student work on the left-hand side of the equlity and modify it to be the right-hand side. Further, as the results of the lack of proving strategies, the student faced difficulties to complete the proof. He should take the sum of the angles, instead of taking the sum of the trigonometric values of the angles.

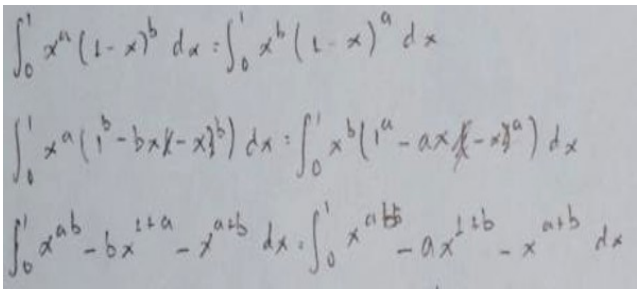


Student's Strategy of Proving a Trigonometric Identity

In this picture, the student failed to prove an equity because his first strategy to work on both side simultaneously had turned into changing the equality to be a difference of the two sides in the previous line, while unfortunately, he did not equalize the difference to zero.



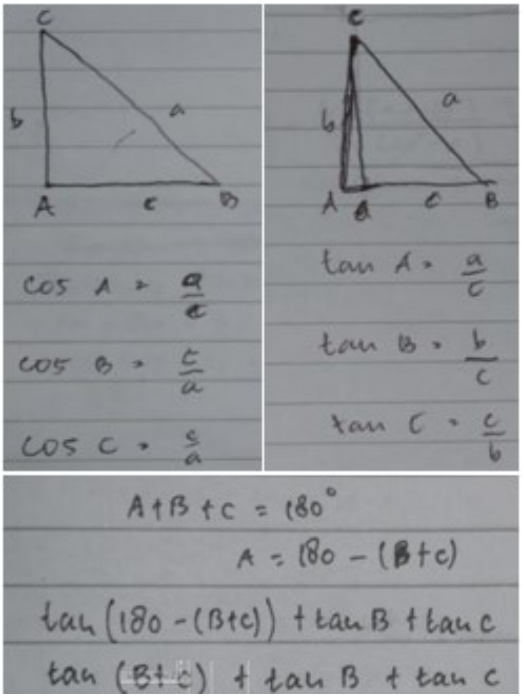
Student's Inappropriate Strategy of Proving



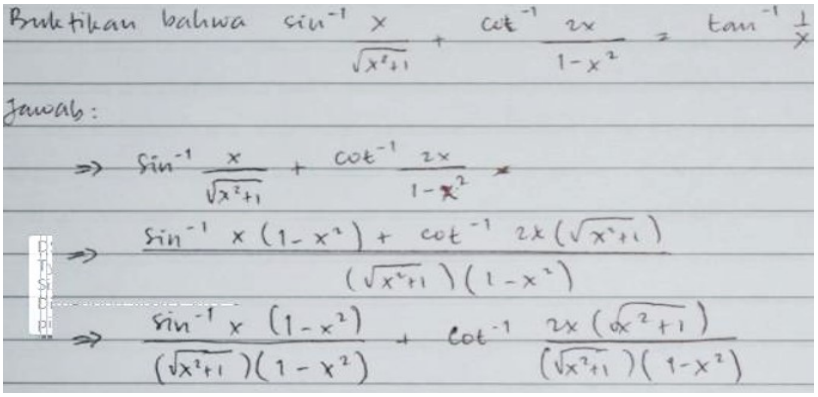
Student's Wrong Binomial Expansion

4.1.4. COMPREHENSION OF MATHEMATICAL CONCEPTS AND PRINCIPLES

The most disturbing difficulty the students face in mathematics proving was their lack of understanding of concepts and principles. They experienced problems in determining the appropriate contexts to employ the principles; therefore falling into an overgeneralization phenomenon. The students showed peculiar thought leading to a strange result. Some students used the expansion of $(a-b)^2$ as the general form for $(a-b)^n$, where it should be expanded using the binomial expansion formula: $(a+b)^n = \sum_{i=0}^{n} C_i^n a^{n-i} b^i$. The trigonometric ratios tan A and cos A were derived based on the generalization of other trigonometric ratios. They failed to understand simple relationships, such as, in trigonometry. For example, in a triangle, they made a false identity, such as tan $(180 - (A + B) = \tan (A + B)$, where it should be tan $(180 - (A + B) = -\tan (A + B)$.



Also, students were confused about the equality and considered $sin\frac{a}{b}$ as the same as $\frac{sina}{b}$, as portrayed in the illustrations.



Student's Confusion about Inverse Trigonometric Function

4.2. DISCUSSION

Students' difficulties in proving seem to root in their lack of understanding of what mathematics proof is and how to construct it. As a discipline built on the deductive-axiomatic system, the students who tend to employ an inductive strategy will experience problems. The domain where the statement applies is sometimes not considered thoughtfully and entirely. Therefore, taking one or two cases is claimed to be enough to show the truth of a statement applying for infinite number of cases. Actually, showing that the statement is correct in several cases can sometimes inspire the students to construct a general deductive proof. This finding is in line with that of Weber's study where verifying a general theorem in one or several cases is considered as an accepted proof [16].

The students still have problems in understanding and using mathematical symbols. The notation difficulty [15] causes them to carelessly use notations or symbols in their sentences; therefore resulting in meaningless expressions. Mathematics is full with symbols and it requires precision in writing or expressing the ideas with symbols. Further, the understanding of symbols will influence the ability to comprehend definitions, propositions, or theorems. Misunderstanding or misconception is another cause of students' difficulty in constructing mathematics proof. As a consequence, the students fail to apply the concepts of principles appropriately [7]. Knowing facts or theorems does not guarantee the correct application of them [16]. Some mistakes performed by the students in the last category found in this present research supports this statement.

By and large, the statements to prove in mathematics are mostly expressed a conditional sentence—an implication. Therefore, for those having weak logical thinking, the proof of a logical statement will be a challenging endeavour. It is found that students encounter significant problems in structuring the proof they construct using logic. In essence, they also show the lack of proving strategies. In solving complex problems, including proving propositions, theorems, or other mathematical statements, someone frequently has several alternative actions, steps, or strategies to employ. However, it should

be realized that only few of them effective in solving the problems. This phenomenon is clearly taking place in the course of proving mathematical statements [16], where one could possess some inferences derived as a strategy for constructing a proof, but most of them are not applicable in the problems encountered. An effective problem solver sometimes has strategic knowledge [16], namely, a heuristic guide to be used to recall actions which are possibly useful to solve a problem or to select appropriate abilities to respond to a challenge. In terms of proving mathematical statements, this quality will be reached by the students if they have been exposed to rich experience of successful proving of mathematical statements.

5. CONCLUSION

It has been found that there were four categories of difficulties experienced by the students in proving statements in mathematics. The four categories were:(1) the use of symbols which was inappropriate; (2) the lack of students' conceptual understanding about mathematics proof; (3) the difficulties in selecting strategies for mathematics proof; and (4) the lack of understanding of concepts and principles in mathematics. It seems that the students in this research need to improve their comprehension of what constitutes a mathematics proof and how to construct a mathematics proof. They might need to be exposed to an activity enriched with various strategies of proving.

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